KINGDOM OF MOROCCO

HASSAN II ACADEMY SCIENCE AND TECHNOLOGY

Advances and perspectives in mathematics

September 8-9, 2022

Académie Hassan II des Sciences et Techniques
Km 4, Avenue Mohammed VI (ex Route des Zaers)
Rabat, Royaume du Maroc

Réalisation : **AGRI-BYS S.A.R.L.**

Achevé d'imprimer : septembre 2022
Imprimerie Lawne : 11, rue Dakar, Océan, 10040-Rabat, Maroc

Sa Majesté le Roi Mohammed VI, que Dieu Le garde,
Protecteur de l'Académie Hassan II
des Sciences et Techniques

Presentation note

On the 8 & 9 september 2022, will take place in Rabat, at the Hassan II Academy of Science and Technology, a workshop on «Advances and perspectives in Mathematics».

Mathematics are renowned worldwide as a central discipline both from the point view of training and research as welle as economic consequences. They have strong interactions with other disciplines such as physics, chemistry computer science, biology, medicine, economy and social sciences.

Organized by the College of Modeling and Information Sciences - Hassan II Academy of Science and Technology - this meeting aimed at bringing together notorious foreign and moroccan, working on various mathematics topics, including analysis, geometry, dynamic systems, probabilities, equations with partial derivatives, interaction with theoretical physics...

Also will participate to this meeting, in addition to confirmrd researchers, Moroccan doctoral students who may be able to set up linkages towards fruitfull collaborations with these researchers, but also to have a vision of the challenges in mathématics for the years to come.

The inaugural conference will be delivered by Prof. Etienne GHYS, Perpetual Secretairy of the french Academy of Sciences, Professor at Ecole Normale Supérieure.

The present meeting is also held within the framework of a task force initiated by the Academy centered on the creation in Morocco of an advanced research structure on Mathematics and their Applications.

Rabat, september 5, 2022

Note de présentation

Les 8 et 9 septembre 2022, se tient à Rabat, au siège de l'Académie Hassan II des Sciences et Techniques, une rencontre sur «Mathématiques, avancées et perspectives».

Les mathématiques sont reconnues au niveau mondial comme étant une discipline centrale tant du point de vue de la formation et de la recherche que des retombées économiques. Elles ont des interactions fortes avec d'autres disciplines telles que la physique, la chimie, l'informatique, la biologie, la médecine, l'économie, les sciences sociales.

Organisée par le Collège Sciences de la Modélisation et de l'Information de l'Académie Hassan II des Sciences et Techniques, cette rencontre a pour objet de réunir d'éminents chercheurs étrangers et marocains, travaillant sur divers axes mathématiques, englobant l'analyse, la géométrie, les systèmes dynamiques, les probabilités, les équations aux dérivées partielles, l'interaction avec la physique théorique...

A cette rencontre participeront, outre des chercheurs confirmés, des doctorants marocains qui pourront tisser des liens pour des collaborations fructueuses avec ces chercheurs, mais aussi d'avoir une vision des chalenges en mathématiques pour les années à venir.

La conférence inaugurale sera donnée par le Professeur Etienne GHYS, Secrétaire Perpétuel de l'Académie des Sciences de France, Professeur à l'Ecole Normale Supérieure.

Cette rencontre s'inscrit également dans le cadre de la réflexion entamée par l'Académie autour de la création au Maroc d'une structure de recherche avancée en Mathématiques et leurs Applications.

Rabat, le 05 Septembre 2022

HASSAN II ACADEMY OF SCIENCES AND TECHNOLOGY

Advances and perspectives in mathematics

SEPTEMBER 8-9, 2022

INAUGURAL CONFERENCE

Etienne Ghys	*Permanent Secretary of the French Academy of Sciences*

PLENARY SPEAKERS

Mohamed Boucetta	*Cadi Ayyad University-Marrakech*
François Delarue	*Côte d'Azur University -France*
Jalal Fadili	*ENSI de Caen - France*
Said Hadd	*University Ibn Zohr, Agadir*
Nordine Mir	*Texas University A&M at Qatar*
Jamal Najim	*Gustave Eiffel University - France*
Nizar Touzi	*Ecole Polytechnique de Paris - France*

ORGANIZED BY

Omar El-Fallah, El Maati Ouhabaz & Youssef Ouknine

Thursday, September 8

Opening Session

9:00-9:30	**Pr. Omar Fassi-Fehri** *Permanent Secretary of the Hassan II Academy of Sciences and technology* **Opening address**

Session I

9h30-10:40	**Inaugural conference : Etienne Ghys** *Permanent Secretary of the French Academy of Sciences and* "Directeur de recherche" CNRS at the ENS in Lyon - *France* **La topologie des courbes algébriques réelles**
10:55-11:20	**Coffe break**
11:20-12:15	**Nordine Mir** *Texas University A&M at Qatar* **On the boundary regularity of holomorphic mappings of positive codimension**
12:30	**Lunch**

Session II

14:00-14:55	**François Delarue** *Côte d'Azur University - France* **Common noise via rearranged stochastic heat equation**
15:10-16:05	**Jalal Fadili** *Ecole Nationale Supérieure d'Ingénieurs de Caen - France* **A dynamical system perspective of optimization in data science**

16:20-16:40	**Coffe break**
16:40-17:35	**Said Hadd** *Ibn Zohr University, Agadir - Maroc* **A functional analytic approach to perturbed deterministic and stochastic equations**

Friday, September 9

Session III

9:00-9:55	**Nizar Touzi** *Ecole Polytechnique de Paris - France* ***M*ean field optimal stopping**
10:10-10:20	**Coffe break**
10:20-10:15	**Mohamed Boucetta** *Cadi-Ayyad University, Marrakech - Maroc* **On the Hermitian structures of the sequence of tangent bundles of an affine manifold endowed with a Riemannian metric**
11:30-12:25	**Jamal Najim** *Gustave Eiffel University, Marne-La-Vallée - France* **Properties of large Lotka-Volterra systems of coupled differential equations with random interactions**
12:40	**Lunch**

Hassan II Academy of Sciences and Technology

Advances and Perspectives in Mathematics

September 08-09, 2022

Abstracts

On the Hermitian structures of the sequence of tangent bundles of an affine manifold endowed with a Riemannian metric

Mohamed Boucetta, Cadi Ayyad University-Marrakech

Let $(M, \nabla, \langle\ ,\ \rangle)$ be a manifold endowed with a flat torsionless connection ∇ and a Riemannian metric $\langle\ ,\ \rangle$ and $(T^kM)_{k\geq 1}$ the sequence of tangent bundles given by $T^kM = T(T^{k-1}M)$ and $T^1M = TM$. We show that, for any $k \geq 1$, T^kM carries a Hermitian structure (J_k, g_k) and a flat torsionless connection ∇^k and when M is a Lie group and $(\nabla, \langle\ ,\ \rangle)$ are left invariant there is a Lie group structure on each T^kM such that (J_k, g_k, ∇^k) are left invariant. It is well-known that (TM, J_1, g_1) is Kähler if and only if $\langle\ ,\ \rangle$ is Hessian, i.e, in each system of affine coordinates $(x_1, \ldots, x_n)$, $\langle \partial_{x_i}, \partial_{x_j}\rangle = \frac{\partial^2 \phi}{\partial_{x_i}\partial_{x_j}}$. Having in mind many generalizations of the Kähler condition introduced recently, we give the conditions on $(\nabla, \langle\ ,\ \rangle)$ so that (TM, J_1, g_1) is balanced, locally conformally balanced, locally conformally Kähler, pluriclosed, Gauduchon, Vaismann or Calabi-Yau with torsion. Moreover, we can control at the level of $(\nabla, \langle\ ,\ \rangle)$ the conditions insuring that some (T^kM, J_k, g_k) or all of them satisfy a generalized Kähler condition. For instance, we show that there are some classes of $(M, \nabla, \langle\ ,\ \rangle)$ such that, for any $k \geq 1$, (T^kM, J_k, g_k) is balanced non-Kähler and Calabi-Yau with torsion. By carefully studying the geometry of $(M, \nabla, \langle\ ,\ \rangle)$, we develop a powerful machinery to build a large classes of generalized Kähler manifolds.

Common noise via rearranged stochastic heat equation

François Delarue, Université Côte d'Azur-France

Motivated by the notion of common noise in mean field games, we here address the construction of a stochastic process with values in the space of probability measures on the real line that features some nice smoothing properties. The strategy is to construct the noise first in the space of random variables as the solution of a stochastic heat equation and then to project on the space of probability measures by rearrangement.

This is a joint work with William Hammersley .

A dynamical system perspective of optimization in data science

Jalal Fadili, Ecole Nationale Supérieure d'Ingènieurs de Caen - France

In this talk, I will discuss and introduce deep insight from the dynamical system perspective to understand the convergence guarantees of first-order algorithms involving inertial features for convex optimization in a Hilbert space setting. Such algorithms are widely popular in various areas of data science (data processing, machine learning, inverse problems, etc.). They can be viewed discrete as time versions of an inertial second-order dynamical system involving different types of dampings (viscous damping, Hessian-driven geometric damping). The dynamical system perspective offers not only a powerful way to understand the geometry underlying the dynamic, but also offers a versatile framework to obtain fast, scalable and new algorithms enjoying nice convergence guarantees (including fast rates). In addition, this framework encompasses known algorithms and dynamics such as the Nesterov-type accelerated gradient methods, and the introduction of time scale factors makes it possible to further accelerate these algorithms. The framework is versatile enough to handle non-smooth and non-convex objectives that are ubiquituous in various applications.

La topologie des courbes algébriques réelles

Etienne Ghys, Ecole normale supérieure de Lyon, France

Une courbe algébrique dans le plan (projectif) réel est définie par une équation polynomiale d'un certain degré k. Pour $k = 1$, il s'agit d'une droite et il n'y a rien à en dire ! Pour $k = 2$, il s'agit d'une conique : un sujet connu depuis l'Antiquité. Pour $k = 3$, c'est Newton qui a dressé une (longue) liste des formes possibles. Dans cet exposé général, je voudrais discuter du cas où le degré k est grand et je voudrais présenter une borne supérieure pour le nombre de types topologiques possibles de courbes (éventuellement singulières) de degré k.

A functional analytic approach to perturbed deterministic and stochastic equations

Said Hadd, University Ibn Zohr, Agadir

In the first part of the talk, on the one hand, we will discuss the concept of the maximal L^p-regularity of perturbed linear equations of the form $\dot{u}+(A+C)u = f$ for any p-integrable function f, where $A : D(A) \subset \mathscr{X} \to \mathscr{X}$ has the maximal L^p-regularity on a Banach space $\mathscr{X}$, and $C : D(A) \to \mathscr{X}$ is a Miyadera-Voigt perturbation and/or $C : \mathscr{X} \to \mathscr{X}_{-1}$ is a Desch-Schappacher perturbation, where $\mathscr{X}_{-1}$ is the extrapolation space of $\mathscr{X}$ w.r.t A. On the other hand, we will use

the sum method to prove the existence of strong solutions of the semilinear equation $\dot{u} + Au + \psi(Cu) = f$ for initial condition $u(0) \in Tr$, the trace space, and for a globally Lipschitz map $\psi : \mathscr{X} \to \mathscr{X}$.
The second part of the talk mainly deals with the well-posedness and Feller property of perturbed stochastic equations with unbounded multiplicative noise

$$dX(t) = AX(t)dt + B(X(t))dW(t), \quad X(0) = \xi, \quad t \geq 0,$$

where A is the generator of an arbitrary $\mathscr{C}_0$-semigroup on Hilbert space $B : D(A) \to L_2(U, H)$ is a nonlinear application given by $B(x) := \mathscr{M}(Kx)$ for $x \in D(A)$, with $\mathscr{M} : H \to L_2(U, H)$ is a nonlinear application and $K : D(A) \to H$ is a linear (not necessarily closed or closable) operator on H. Here $L_2(U, H)$ denotes the space of all Hilbert-Schmidt operators acting between U and H, and $(W(t))_{t\geq 0}$ is a Q-Wiener process on a filtered probability space $(\Omega, \mathscr{F}, \mathbb{P})$. We conclude the talk with an application of the maximal regularity to a class of stochastic integro-differential equations.

The talk combines works in collaboration with A. Amansag, H. Bounit and A. Driouich, M. Fkirine and F.Z. Lahbiri from the Department of Mathematics, Faculty of Sciences, Ibn Zohr University, Agadir, Morocco. A. Amansag, H. Bounit, A. Driouich, and S. Hadd, On the maximal regularity for perturbed autonomous and non-autonomous evolution equations. J. Evol. Equ., **20**, 165–190 (2020).

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[2] S. Hadd, The sum method to stochastic integro-differential equations. Preprint.

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On the boundary regularity of holomorphic mappings of positive codimension

Nordine Mir, Texas University A&M at Qatar

We discuss the C^∞ regularity problem for CR maps $h\colon M \to M'$ between C^∞-smooth CR submanifolds M, M' embedded in complex spaces of possibly different dimensions. For real hypersurfaces $M \subset \mathcal{C}^{n+1}$ and $\mathcal{C}^{n'+1}$ with $n' > n \geq 1$ and M strongly pseudoconvex, we prove that every CR transversal map of class $C^{n'-n+1}$ that is nowhere C^∞ on some non-empty open subset of M must send this open subset to the set of D'Angelo infinite points of M' . As

a consequence we establish the following boundary regularity result for proper holomorphic maps in positive codimension: given $\Omega \subset \mathcal{C}^{n+1}$ and $\Omega' \subset \mathcal{C}^{n'+1}$ pseudoconvex domains with smooth boundaries $\partial\Omega$ and $\partial\Omega'$ both of D'Angelo finite type, $n' > n \geq 1$, any proper holomorphic map $h\colon \Omega \to \Omega'$ that extends $C^{n'-n+1}$-smoothly up to $\partial\Omega$ must be C^∞-smooth on a dense open subset of $\partial\Omega$. We shall also discuss more general regularity results for CR submanifolds of higher codimension, in which the target manifolds are allowed to be even of D'Angelo infinite type.

Properties of large Lotka-Volterra systems of coupled differential equations with random interactions

Jamal Najim, Université Paris-Est Marne-La-Vallée - France

Lotka-Volterra (LV) systems of coupled differential equations are widely used in mathematical biology and theoretical ecology to model populations with interactions. In the case of a large system, the random interactions form a large random matrix the statistical properties of which reflect (to some extent) some properties of the underlying biological system.

In this talk, we will present some properties of such large LV systems (existence of a unique equilibrium, its positivity, and its stability properties) for some standard random matrix models (circular, elliptic, sparse).

Based on joint works with Akjouj, Clenet, El Ferchichi, Massol.

Mean field optimal stopping

Nizar Touzi, Ecole Polytechnique de Paris - France

We study the optimal stopping problem of McKean-Vlasov diffusions when the criterion is a function of the law of the stopped process. A remarkable new feature in this setting is that the stopping time also impacts the dynamics of the stopped process through the dependence of the coefficients on the law. The mean field stopping problem is introduced in weak formulation in terms of the joint marginal law of the stopped underlying process and the survival process. Using the dynamic programming approach, we provide a characterization of the value function as the unique viscosity solution of the corresponding dynamic programming equation on the Wasserstein space. Under additional smoothness condition, we provide a verification result which characterizes the nature of optimal stopping policies, highlighting the crucial need to randomized stopping. Finally, we the convergence of the the finite population multiple optimal stopping problem to the corresponding mean field optimal stopping limit. These results of propagation of chaos are proved by adapting the Barles-Souganidis monotonic scheme method to the present context.

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