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Edited by The Hassan II Academy of Science and Technology of Morocco

Contents

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FOREWORD

In this issue of Frontiers in Science and Engineering International Journal, four papers are edited, two of them are dealing with mathematics and applications while the other two are related to catastrophic natural and societal-induced environmental hazards.

The first paper entitled “Structure of topological algebras and lemma of Müldner is adressing a gap in Müldners proof, concerning the structure of general complete topological algebras while the second paper entitled “Classification of spatio-temporal systems: Concept of domination”, is an overview on spatio-temporal (distributed) systems classification.

The third paper entitled “Natural and societal-induced environmental hazards: integrate long–term interdisciplinary research strategy for developing countries deals with science of natural and societal-induced hazards and disaster risk and describes, with details, a research multidisciplinary initiative (FONDAP Programs) highlighting long-term results regarding first world class publications, that may serve as an example for building natural and S-IHEs investigative and design public policy strategies in other developing countries.

The fourth paper entitled “Revisiting the Pasteur Quadrant, Post-normal Science and Strategies for Research on Natural Hazards and Disasters”, presents the concept of quadrants with special emphasis while dealing with catastrophic events.

Prof. Driss OUAZAR
Chief Editor

Structure of topological algebras and a lemma of Müldner

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Abstract

We redress a gap in Müldner's proof, concerning the structure of general complete topological algebras. We also examine the strucure of locally A-F-normed ones.

Key words: topological algebra, projective limit, A-F-seminorm.
S.C.N. 46H05, 46H20

1/ Introduction and preliminaries. The results of Müldner (Theorem 1 and Theorem 2, [9]) are based on a lemma the proof of which is not valid. We exhibit errors and give a correct and simple proof (Lemma 2.3). This is for general topological algebras. On another hand, we introduce a subclass of the latter; the one of the so called locally A-F-normed algebras. For these, one can argue as in the now classical case of m-convex ones; and give a more precice statement (Proposition 3.7).

A topological algebra is an algebra A over $\mathbb{K}$ (R or $\mathbb{C}$) endowed with a topological vector space topology τ, for which multiplication is separately continuous. If the map $(x, y) \longmapsto xy$ is continuous (in both variables), then A is said to be with continuous multiplication. We say that a unital topological algebra is a Q-algebra if the set $G(A)$ of its invertible elements is open.

Let (A, τ) be a locally convex algebra ($l.c.a.$), with a separately continuous multiplication, whose topology τ is given by a family $(p_\lambda)_\lambda$ of seminorms. The algebra (A, τ) is said to be locally A-convex ([5], [6]) if, for every x and every λ, there is $M(x, \lambda) > 0$ such that

$$\max\,[p_\lambda(xy), p_\lambda(yx)] \leq M(x, \lambda)p_\lambda(y); \forall y \in A.$$

In the case of a single space norm, $(A, \|.\|)$ is called an A-normed algebra. If $M(x, \lambda) = M(x)$ depends only on x, we say that (A, τ) is a locally uniformly A-convex algebra [6]. If it happens that, for every λ,

$$p_\lambda(xy) \leq p_\lambda(x)p_\lambda(y); \forall x, y \in A,$$

then (A, τ) is named a locally m-convex algebra ($l.m.c.a.$; [7], see also [2] and [6]). Recall also that a $l.c.a.$ has a continuous multiplication if, for evry λ, there is λ' such that $p_\lambda(xy) \leq p_{\lambda'}(x)p_{\lambda'}(y); \forall x, y \in A$.

Let A be a complex vector space and $p \in]0,1]$. A map $|.|_p : A \longrightarrow \mathbb{R}_+$ is said to be a p-seminorm if

(i) $|x+y|_p \leq |x|_p + |y|_p$, for every x and every y in A.

(ii) $|\mu x|_p = |\mu|^p |x|$, for every μ in $\mathbb{C}$ and every x in A.

It is a p-norm (and is denoted $\|.\|_p$), if moreover it satisfies

(iii) $|x|_p = 0$ implies $x = 0$.

Let (A, τ) be a Hausdorff locally pseudo- convex space and $(|.|_i)_i$ a family of p_i-seminorms defining its topology. If A is endowed with an algebra structure such that multiplication is separately continuous, then $\left(A, (|.|_i)_i\right)$ is named a locally pseudo- convex algebra.

2. On a lemma of Müldner. In [8], Müldner stated the following results

Theorem 2.1 ([9], Theorem 1)**.** Every complete topological (Hausdorff) algebra is isomorphic to a projective limit of F-algebras (i.e., complete metrizable algebras).

Theorem 2.2 ([9], Theorem 2)**..** Every complete locally convex (topological Hausdorff) algebra is isomorphic to a projective limit of B_0 algebras (i.e., complete metrizable locally convex algebras).

The proofs rely on the following lemma. But the proof of the latter is not correct. Let (A, τ) be a topological algebra and $\mathcal{V} = \mathcal{V}(0)$ a basis of neighborhoods of the origin, made of closed balanced ones. Put $N = \cap V$, V running over $\mathcal{V}$.

Lemma 2.3 ([9],Lemma). Let (E, τ) be topological algebra over $\mathbb{K}$ ($\mathbb{R}$ or $\mathbb{C}$). If it is with a continous product, then the subset N is a closed ideal in (E, τ).

Remark 2.4. The proof given in [9] is not correct. Indeed, from the very beginning, the author takes $\left(\alpha^2+\beta^2\right)^{-1}$ for every α and β in the complex field $\mathbb{C}$. Also, it seems that it has been considered that

$$\left|\frac{\alpha}{\alpha^2+\beta^2}\right| \leq 1; \forall \left(\alpha, \beta\right) \in \mathbb{C}^2;$$

which is not granted. Now, here is a proof of the lemma above.

Proof (of the lemma)**.** We first show that N is a vector space. The continuity of the addition immediately implies that $x+y \in N$, for any x, y in N. For multiplication by scalars, it is enough to consider positive real numbers (see below). So let $x \in N$, $\alpha > 0$ and denote by $\mathcal{E}(\alpha)$ the entire part of α. One has $\alpha\left(\mathcal{E}(\alpha)+1\right)^{-1} \leq 1$. Hence $\alpha x \in \left(\mathcal{E}(\alpha)+1\right)V$, for every V in $\mathcal{V}$. Now

let U be an arbitrary element in $\mathcal{V}$. If $\mathcal{E}(\alpha)$ is even, take V_α in $\mathcal{V}$ such that $\left(\mathcal{E}(\alpha)+2\right)V \subset U$. Then

$$\alpha x \in \left(\mathcal{E}(\alpha)+1\right)V_\alpha \subset \left(\mathcal{E}(\alpha)+2\right)V_\alpha \subset U.$$

If $\mathcal{E}(\alpha)$ is odd, take V_α in $\mathcal{V}$ such that $\left(\mathcal{E}(\alpha)+1\right)V_\alpha \subset U$. For negative numbers, just retain that the elements of V are balanced. Finally, use $\alpha x = ax + ibx$, for a complex number $a+ib$.

To show that N is an ideal, we reproduce the argument of Müldner. Let x be a non zero element x in N, and U in $\mathcal{V}$. Choose V such that $V^2 \subset U$. For any y in $\mathcal{E}$, there is an $\alpha > 0$ such that $\alpha y \in V$ (absorbness). But by the preceding, $\alpha^{-1}x \in V$. Hence

$$xy = \alpha^{-1}x\alpha y \in V^2 \subset U.$$

Remark 2.5. Mati Abel has already provided a proof ([1], Lemma 2.1, p. 11). Our's is shorter and uses less notions.

3. Locally A-F-normed algebras. A-F-normed algebras have been studied in [4]. Here we introduce the new class of locally A-F-normed algebras, and its sublass of locally uniformly A-F-normed ones. For the latter, one can argue as in the classical way and obtain a more precise structure result. For the notion of an F-seminorm, in a vector space, the reader is refereed to [10]. The topology of a topological vector space can be defined by a family of F-seminorms (cf. [10]). Recall that an F-seminorm is a map $|.| : E \longrightarrow \mathbb{R}_+$, on a vector space such that

(i) $|x+y| \leq |x| + |y|$, for every x and every y in A.
(ii) $|\alpha x| \leq |x|$, for every α in $\mathbb{C}$ with $|\alpha| \leq 1$, and every x in A.
(iii) If $|x_n| \longrightarrow 0$, then $|\alpha x_n| \longrightarrow 0$ for every α in $\mathbb{C}$.
(iv) If $|\alpha_n| \longrightarrow 0$, then $|\alpha_n x| \longrightarrow 0$ for every x in E.

Definition 3.1. A locally A-F-normed algebra is a topological vector space, endowed with an algebra structure, the toplogy of which is given by a family $(|.|_i)_i$ of F-seminorms such that, for every x and every λ, there is $M(x,\lambda) > 0$ such that

$$\max\left[p_\lambda(\frac{x}{M(x,\lambda)}y), p_\lambda(y\frac{x}{M(x,\lambda)})\right] \leq p_\lambda(y); \forall y \in A.$$

In the case of a single F-seminorm, $(A, |.|)$ is called an A-F-seminormed algebra. If $M(x,\lambda) = M(x)$ depends only on x, we say that $(A, (|.|_i)_i)$ is a locally uniformly A-F-normed algebra.

Example 3.2. Every p-Banach algebra ([12], [13]) is a complete A-F-normed algebra. The converse is not true (Example 3.2).

Example 3.3. Let $\mathcal{M}[0,1]$ be the algebra of complex measurable functions on the the segment $[0,1]$ and put $N = \{f \in \mathcal{M}[0,1] : f = 0, E.e.\}$. Consider the quotient algebra $E = \mathcal{M}[0,1] \diagup N$, endowed with the topology τ of convergence in measure. Then (E,τ) is a complete metrizable topological algebra[11]**.** Its topology can be defined by the F-norm q given by

$$q(f) = \inf\{\delta + p_\delta(f) : \delta > 0\}$$

where

$$p_\delta(f) = \mu(\{x \in [0,1] : |f(x)| \geq \delta\}),$$

μ being the Lebesgue measure on $[0,1]$. It is not a p-Banach algebra. However, the product is continuous.

Example 3.4. Take $E = \mathcal{M}[0,1]$ of the previous example. Any standard cartesian product ΠE_i, with $E_i = E$ for every i, is a complete $l.A\text{-}F\text{-}n.a.$

Example 3.5. Let $C_b(\mathbb{R})$ be the algebra of complex continuous bounded functions on the real field $\mathbb{R}$ with the usual pointwise operations. Denote by $C_0^+(\mathbb{R})$ the srictly positive elements of $C_b(\mathbb{R})$. Consider the family $\{p_\varphi : \varphi \in C_0^+(\mathbb{R})\}$ of seminorms given by

$$p_\varphi(f) = \sup\{f(x)\varphi(x) : x \in \mathbb{R}\}; f \in C_b(\mathbb{R}).$$

The space $(C_b(\mathbb{R}),)$ is a complete $l.u\text{-}A\text{-}c.a.$ It is not a $l.m.c.a.$ ([5]), nor a Q-algebra.

Example 3.5. Take the standard cartesian product $C_b(\mathbb{R}) \times E$, where E is the algebra in Example 3.3. Then $C_b(\mathbb{R}) \times E$ is a $l.u\text{-}A\text{-}F\text{-}n.a.$which is not a locally pseudo-convex algebra.

Example 3.6. Let $C(\mathbb{R})$ be the algebra of complex continuous functions on the real field, with the usual pointewise operations and the topology τ given by the family $(|.|_K)_K$ of seminorms; where $|f|_K = \sup\{|f(x)| : x \in K\}$, K running on the collection of compact subsets of $\mathbb{R}$. Then $(C(\mathbb{R}),\tau)$ is a complete m-convex algebra. The standard cartesian product $C(\mathbb{R}) \times E$, where E is the algebra of Example 3.3 , is a $l\text{-}A\text{-}F\text{-}n.a.$ which is not a uniformly $A\text{-}F$-normed one.

Proposition 3.7. Let $(A, (|.|_\lambda)_\lambda)$ be a complete $l\text{-}A\text{-}F\text{-}n\text{-}a$. If the product is continuous with respect to each F-seminorm, then, it is a projective limit of complete $A\text{-}F$-normed algebras.

Proof. First, observe that there is $N(x,\lambda) > 0$ such that

$$\max[p_\lambda(xy), p_\lambda(yx)] \le N(x,\lambda)p_\lambda(y); \forall y \in A.$$

Indeed,

$$p_\lambda(xy) = p_\lambda(\frac{x}{M(x,\lambda)}M(x,\lambda)y) \le p_\lambda(M(x,\lambda)y) \le [\mathcal{E}M(x,\lambda)+1]\,p_\lambda(y).$$

Thus $N_\lambda = \{x : p_\lambda(x) = 0\}$ is a closed bilateral ideal. Hense $E_\lambda = E/N_\lambda$, endowed by the F-norm $\overline{p_\lambda}$ given by $\overline{p_\lambda}(\overline{x}_\lambda) = p_\lambda(x)$, is an $A\text{-}F$-normed algebra. Now, the product being continuous with respect to p_λ, the completion of $(E_\lambda, \overline{p_\lambda})$ is a complete $A\text{-}F$-normed algebra. The rest is a matter of standard techniques concerning projective limits.

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Classification of spatio-temporal systems: Concept of domination

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Abstract. *This work is an overview on spatio-temporal (distributed) systems classification. More precisely, it concerns the notion of domination for a class of distributed parameter systems. We explore the possibility of classification of such systems based on the choice of input and output operators. The systems to be compared may have different dynamics. We give appropriate definitions of the weak and exact domination, the main properties and characterization results for general controlled systems and input operators. By duality, similar results are deduced for observed systems and output operators. The cases of multi-actuators and multi-sensors are considered. Relationship with the notion of remediability is also explored. Applications of the results to the case of parabolic and hyperbolic systems are considered.*

Key words: Distributed Parameter Systems, Domination, Actuators, Sensors.

1. Introduction

The study of real systems often lead to introduce new approaches and tools which allow more knowledge of the system evolution. Furthermore it can help in the prediction of its evolution or in the choice of controls. Systems theory remain the best context for this exploration. For this purpose the usual scheme "$Modelling \to Analysis \to Control$" needs to focus on the analysis step. It consists in a set of various concepts which have been explored from the 50's, first for lumped systems and later for distributed parameter systems. Among these concepts are the usual controllability, observability, stability and so on. These concepts are different when the systems are distributed in space and time. Furthermore the space variable brings various new ideas which do not exist in the case of lumped systems. Additionally in real distributed systems, actuators and sensors have a space existence defined by their location in the space domain and their space distribution, i.e. how the actuators act really in space. See [2, 4, 5, 7, 13, 14, 26].

Considering applications in ecology, environment, etc. the study of such concepts is not adapted to the whole space. That is why the regional approach in distributed systems analysis has been considered. Various works have been developed and enhance the meaning of such notions, see [8, 12, 14] and [17, 18, 25] and the references therein. Motivating applications have been considered. The stabilizability is considered when one has to find a control which makes stable a system. This also may have a regional sense. By duality the detectability has also be considered and widely explored. Detection of unknown sources has been also studied by various researchers. It consists to reconstruct a space disturbance using measurements given by an output function. The sensors which allow such reconstruction have been called *spy sensors*, see [15, 19, 20, 27].

All these notions together with new biogeographical phenomena have naturally led to what is called spreadability. It consists to study distributed systems in which a given property increase (or decrease) in space, as it can be seen in vegetation dynamics, desertification, pollution, cancer disease, etc. Various degrees of spreadability have been considered and a wide literature published. To this context spray control approach has been considered, see [9, 16]. As a consequence the spreadability has led to the space compensation of a disturbance. When a space disturbance can be spatially controlled (or eliminated) thus the system is said to be remediable. The disturbance effect is observed via sensors and the space compensation is done via actuators, both have to be made precise and correctly located. Remediability has been considered both in finite time and asymptotically. Furthermore some authors have considered minimum energy remediability, that is to say the minimum cost for a space compensation, [22, 23, 24, 30].

In this paper we present an extension of the domination concept to a class of continuous time distributed systems. The comparison is done for systems which have not necessarily the same dynammics. We first study the problem of domination for controlled systems. We give the main characterization results and properties. By duality, we obtain analogous results for observed systems and output operators.
Then, we consider in the next section, the case of parabolic systems (diffusion systems), with multi-actuators and multi-sensors. The characterizations and results depend naturally on the corresponding controllability and observability matrices. Applications to one and two space dimension are examined. Then we consider the case of a class of hyperbolic systems. We show that the results and properties remain similar.

In distributed parameter systems, the space plays an important role. Moreover the choice of the space location for the control is tremendously important. That means there are controls which may be more efficient than others because they have been conveniently located or chosen. This opens a wide field of classification of input operators with respect to a given objective. This is what we call domination concept in distributed parameter systems analysis, and is the purpose of this paper. We introduce the mathematical context for the considered development. We introduce exact and weak domination, emphasizing input and output operators (sensors and actuators). We show how it can be applied to both parabolic and hyperbolic systems. Illustrative applications are also considered.

Let us first consider, without loss of generality, a class of linear disturbed parameter systems described by the following state equation :

$$\left\{\begin{array}{lcl}\dot{z}(t) & = & Az(t)+f(t)+Bu(t) \quad ; \quad 0<t<T \\ \\ z(0) & = & z_0 \in Z\end{array}\right. \tag{1.1}$$

where A generates a strongly continuous semi-group (denoted s.c.s.g. in what follows) $(S(t))_{t\geq 0}$ on the state Z. $B \in \mathcal{L}(U,Z)$, $f \in L^2(0,T;Z)$ is a disturbance, $u \in L^2(0,T;U)$ is a control term; Z and U are respectively the state and the control spaces, assumed to be Hilbert spaces. The system (1.1) is augmented with the following output equation

$$y(t) = Cz(t) \tag{1.2}$$

with $C \in \mathcal{L}(Z,Y)$, Y is a Hilbert observation space. The operator A describes the dynamics of the system, the operators B and C are respectively the input and output operators. In the case where the system (1.2) is autonomous (said the normal case), i.e. $f \equiv 0$ and $u = 0$, we have $z(t) = S(t)z_0$. Thus the observation is normal and given by

$$y_0(t) = CS(t)z_0$$

But if $f \neq 0$, we have

$$y(t) = CS(t)z_0 + \int_0^t CS(t-s)f(s)ds$$

Generally, $y \neq y_0$. The disturbance f is usually unknown (totally or partially). The problem of detection consists to study, with respect to the output operator C (sensors), the possibility to reconstruct any disturbance f, from the corresponding observation only.
This problem was explored for various types of systems (parabolic or hyperbolic, continuous or discrete time, global or regional cases) [10, 11, 20, 21, 27, 37, 42]. However, the knowledge of f is not sufficient. One has to study the possibility to bring the observation, at the final time T, to its normal state with a convenient control applied via the control operator B. This problem can be formulated as follows.
For all $f \in L^2(0,T;Z)$, there exists a control $u \in L^2(0,T;U)$ such that

$$CH_{_B}u + CRf = 0 \tag{1.3}$$

where H_B and R are the operators defined by

$$\begin{array}{rcccl} H_B & : & L^2(0,T;U) & \longrightarrow & Z \\ & & u & \longrightarrow & H_B u = \displaystyle\int_0^T S(T-t)Bu(t)dt \end{array} \tag{1.4}$$

$$\begin{array}{rcccl} R & : & L^2(0,T;Z) & \longrightarrow & Z \\ & & f & \longrightarrow & Rf = \displaystyle\int_0^T S(T-t)f(t)dt \end{array} \tag{1.5}$$

The relation (1.3) means that the control effect has compensated the disturbance. This defines the notion of remediability. This notion has also been studied in the exact case, as well as the weak one, for parabolic or hyperbolic systems, continuous or discrete time, global and regional cases. The case where the observation is affected by an error has been considered. A wide literature exists, see [22, 23, 24, 28, 37, 42] and the references therein.

Consider now the particular case where the disturbance f is given by $f(t) = Gv(t)$, with $G \in \mathcal{L}(W, Z)$, $v \in L^2(0,T;W)$; W is a Hilbert space, the system (1.1) becomes

$$\begin{cases} \dot{z}(t) = Az(t) + Gv(t) + Bu(t) \quad ; \quad 0 < t < T \\ z(0) = z_0 \in Z \end{cases} \tag{1.6}$$

This is a usual formulation in control theory, and means that the disturbance is due to accidental or voluntary actions. It is often used in control theory and describes more precisely the nature of the disturbance $Gv(.)$, its location, its spatial distribution and its intensity. Hence, for the associated compensation problem, the choice of the control operator B will depend obviously on G.
In this section, and in order to simplify, we consider the case where $C = I$. The exact remediability problem is stated as follows :

Let H_B be the operator defined in (1.4) *and H_G defined by*

$$\begin{array}{rcccl} H_G & : & L^2(0,T;W) & \longrightarrow & Z \\ & & v & \longrightarrow & H_G v = \displaystyle\int_0^T S(T-s)Gv(s)ds \end{array} \tag{1.7}$$

For $v \in L^2(0,T;W)$, does a control $u \in L^2(0,T;U)$ such that

$$H_G v + H_B u = 0 \tag{1.8}$$

exist ?

The exact remediability is then equivalent to

$$Im[H_G] \subset Im[H_B] \tag{1.9}$$

and the weak remediability is equivalent to

$$\overline{Im[H_G]} \subset \overline{Im[H_B]} \tag{1.10}$$

These inclusions mean respectively that the operator B is stronger than G exactly and weakly, in the sense that B is able to compensate exactly (respectively weakly) the effect of any action due to the operator G.

Equivalently in the usual case where B and G correspond respectively to actuators $(\Omega_i, h_i)_{1\leq i\leq p}$ and $(D_j, h_j)_{1\leq j\leq q}$, then actuators $(\Omega_i, h_i)_{1\leq i\leq p}$ are more efficient than $(D_j, h_j)_{1\leq j\leq q}$ and dominate them. This is the origin of the notion of domination introduced and developed firstly for continuous lumped and distributed systems having the same dynamics, see [30, 32, 42]. It consists to study the possibility of exact or weak comparison (or classification) of input operators (actuators), and by duality for output operators (sensors).

The number of actuators (or sensors) is not a sufficient condition (criterion) for the domination. Indeed, one actuator (sensor) may dominate several others. On an other hand, concerning the dual notions of controllability and observability, for a given operators B and C, the main problem is mainly to know if a system is controllable (or observable) and then how to find the optimal control ensuring to reach a desired state z_d (or reconstruct the initial state z_0). The problem of classification and comparison of input (output) operators themselves was not really studied. The notion of domination allows to make a comparison of input and output operators, and then to make a better choice. It was initially introduced and studied for systems having the same dynamics.

2. Exact and weak domination of distributed systems

2.1. Problem statement and definitions

We consider the following linear distributed systems

$$(\mathcal{S}_1)\ \begin{cases} \dot{z}_1(t) = A_1 z_1(t) + B_1 u_1(t)\ ;\ 0 < t < T \\ \\ z_1(0) = z_{1,0} \in Z \end{cases} \tag{2.1}$$

$$(\mathcal{S}_2)\ \begin{cases} \dot{z}_2(t) = A_2 z_2(t) + B_2 u_2(t)\ ;\ 0 < t < T \\ \\ z_2(0) = z_{2,0} \in Z \end{cases} \tag{2.2}$$

where, for $i = 1, 2$; A_i is a linear operator generating a s.c.s.g. $(S_i(t))_{t\geq 0}$ on the state space Z. $B_i \in \mathcal{L}(U_i, Z)$, $u_i \in L^2(0, T; U_i)$; U_i is a control space. The systems (S_1) and (S_2) are respectively augmented with the output equations

$$(\mathcal{E}_i)\quad y_i(t) = Cz_i(t) \text{ for } i = 1, 2\ ;\ C \in \mathcal{L}(Z, Y)$$

The state of the system $(\mathcal{S}_i)$ at the final time T is given by

$$z_i(T) = S_i(T) z_{i,0} + H_{B_i} u_i \tag{2.3}$$

where

$$\begin{array}{llll} H_{B_i}: & L^2(0, T; U_i) & \longrightarrow & Z \\ & u_i & \longrightarrow & \displaystyle\int_0^T S_i(T - s) B_i u_i(s) ds \end{array} \tag{2.4}$$

The corresponding observation at time T is given by

$$y_i(T) = CS_i(T) z_{i,0} + CH_{B_i} u_i \tag{2.5}$$

The purpose is to study a possible comparison of systems $(\mathcal{S}_1)$ and $(\mathcal{S}_2)$ (or the input operators B_1 and B_2 if $A_1 = A_2$) with respect to the output operator C. This comparison will be based on the dynamics A_1 and A_2, the control operators B_1, B_2 and the observation operator C. Without loss of generality, one can assume that $z_{1,0} = z_{2,0} \equiv 0$. We introduce hereafter the corresponding notion of domination.

Definition 1. *We say that*

i. The system $(\mathcal{S}_1)$ dominates the system $(\mathcal{S}_2)$ (or equivalently the pair (A_1, B_1) dominates (A_2, B_2)) exactly on $[0, T]$, with respect to the operator C, if

$$Im(CH_{B_2}) \subset Im(CH_{B_1})$$

ii. The system $(\mathcal{S}_1)$ dominates the system $(\mathcal{S}_2)$ (or equivalently the pair (A_1, B_1) dominates (A_2, B_2)) weakly on $[0, T]$, with respect to the operator C, if

$$\overline{Im(CH_{B_2})} \subset \overline{Im(CH_{B_1})}$$

In this situation, we note respectively

$$(A_2, B_2) \underset{C}{\leq} (A_1, B_1)\ \ and\ \ (A_2, B_2) \underset{C}{\lesssim} (A_1, B_1)$$

Furthermore we have the following properties and remarks.

1. Obviously, the exact domination with respect to an output operator C, implies the weak one with respect to C. The converse is not true, this is shown in [32, 42] in the case where $A_1 = A_2$ and $C = I$.

2. If the system $(\mathcal{S}_1)$ is controllable exactly (respectively weakly), or equivalently

$$Im(H_{B_1}) = Z \text{ (respectively } \overline{Im(H_{B_2})} = Z)$$

   then the system $(\mathcal{S}_1)$ dominates exactly (respectively weakly) any system $(\mathcal{S}_2)$, with respect to any output operator C.

3. In the case where $A_1 = A_2$, and the system $(\mathcal{S}_1)$ dominates the system $(\mathcal{S}_2)$ exactly (respectively weakly), we say simply that B_1 dominates B_2 exactly (respectively weakly). Then, we note

$$B_1 \underset{C}{\leq} B_2 \ \ (respectively\ B_1 \underset{C}{\lesssim} B_2).$$

   Hence, one can consider a single system with two inputs as follows

$$(\mathcal{S}) \quad \begin{cases} \dot{z}(t) = Az(t) + B_1u_1(t) + B_2u_2(t); \ 0 < t < T \\ \\ z(0) = z_0 \in Z \end{cases} \tag{2.6}$$

   augmented with an output equation

$$(\mathcal{E}) \ \ y(t) = Cz(t)$$

   In this case, the domination of control operators B_1 and B_2, with respect to the observation operator C, is similar. The definitions and results remain the same.

4. The exact or weak domination of systems is a transitive and reflexive relation, but it is not antisymmetric. Thus, for example in the case where $A_1 = A_2$, for any non-zero operator $B_1 \neq 0$ and $\alpha \neq 0$, we have $Im(CH_{B_1})) = Im(CH_{\alpha B_1})$, even if $B_1 \neq \alpha B_1$ for $\alpha \neq 1$.

5. For the relationship with the notion of remediability, we consider without loss of generality, a class of linear distributed systems described by the following state equation

$$\begin{cases} \dot{z}(t) = Az(t) + Bu(t) + d(t) \ ; \ 0 < t < T \\ z(0) = z_0 \end{cases} \tag{2.7}$$

   where $d \in L^2(0, T; Z)$ is a known or unknown disturbance. The system (2.7) is augmented with the following output equation

$$y(t) = Cz(t) \tag{2.8}$$

   The state z of the system at time T is given by

$$z(T) = S(T)z_0 + H_Bu + Rd$$

   where H_B is defined in (1.4) and

$$Rd = \int_0^T S(T-s)d(s)ds$$

   If the system (2.7), augmented with (2.8), is exactly (respectively weakly) remediable on $[0, T]$, or equivalently $Im(CR) \subset Im(CH_B)$ (respectively $\overline{Im(CR)} \subset \overline{Im(CH_B)}$), then B dominates any operator B_2 exactly (respectively weakly) with respect to the operator C.

6. In the case where $C = I$ and $A_1 = A_2$, one retrieves the particular notion of domination as considered in [32, 42].

We give hereafter characterization results concerning the exact and weak domination.

2.2. Characterizations

The following result gives a characterization of the exact domination with respect to the output operator C.

Proposition 2. *The following properties are equivalent*

i. The system $(\mathcal{S}_1)$ *dominates exactly the system* $(\mathcal{S}_2)$ *with respect to the operator* C.

ii. For any $u_2 \in L^2(0,T;U_2)$*, there exists* $u_1 \in L^2(0,T;U_1)$ *such that*

$$CH_{B_1}u_1 + CH_{B_2}u_2 = 0 \tag{2.9}$$

iii. There exists $\gamma > 0$ *such that for any* $\theta \in Y'$*, we have*

$$||B_2^*S_2^*(T-.)C^*\theta||_{L^2(0,T:U_2)} \leq \gamma||B_1^*S_1^*(T-.)C^*\theta||_{L^2(0,T:U_1)} \tag{2.10}$$

Proof.
The equivalence between i) and ii) derives from the definition.
The equivalence between ii) and iii) is a consequence of the fact that if X, Y and Z are Banach spaces; $P \in \mathcal{L}(X,Z)$ and $Q \in \mathcal{L}(Y,Z)$, then

$$Im(P) \subset Im(Q)$$

if and only if, there exists $\gamma > 0$ such that for any $z^* \in Z^{'}$, we have

$$||P^*z^*||_{X'} \leq \gamma||Q^*z^*||_{Y'}$$

where X', Y' and Z' are respectively the dual spaces of X, Y and Z. □

Concerning the weak case, we have the following characterization result.

Proposition 3.
The system (S_1) *dominates the system* (S_2) *weakly, with respect to* C*, if and only if*

$$\ker[B_1^*S_1^*(.)C^*] \subset \ker[B_2^*S_2^*(.)C^*] \tag{2.11}$$

Proof. Derives from the definition and the fact that

$$\overline{Im(CH_{B_2})} \subset \overline{Im(CH_{B_1})}$$

is equivalent to

$$\ker[(CH_{B_1})^*] \subset \ker[(CH_{B_2})^*].$$

□

It is well known that the choice of the input and output operators play an important role in the controllability and observability of a system [1, 2, 3, 5, 6, 7, 13]. Here also, the domination for controlled systems, with respect to an output operator C, depends on the dynamics A_i and particularly on the choice of the control operators B_i. However, even if $B_1 = B_2 = B$ (with the same actuator), the pair (A_1, B) may dominates (A_2, B). This is illustrated in the following example.

Example 4. *We consider the system described by the one dimension equation*

$$\begin{cases} \dfrac{\partial z(x,t)}{\partial t} & = & \alpha\dfrac{\partial^2 z(x,t)}{\partial x^2} + \beta z(x,t) + g(x)u(t) &]0,1[\times]0,T[\\ z(0,t) & = & z(1,t) = 0 &]0,T[\\ z(x,0) & = & z_0(x) &]0,1[\end{cases}$$

The operator $M(\alpha,\beta)=\alpha\dfrac{\partial^2}{\partial x^2}+\beta I$ *generates the s.c.s.g.* $(S_{(\alpha,\beta)}(t))_{t\geqslant 0}$ *defined by*

$$S_{(\alpha,\beta)}(t)z=\sum_{n=1}^{+\infty}e^{(\beta-\alpha n^2\pi^2)t}\langle z,\varphi_n\rangle\varphi_n$$

where $(\varphi_n)_n$*, with* $\varphi_n(x)=\sqrt{2}\sin(n\pi x)$*, is a complete system of eigenfunctions of* $M(\alpha,\beta)$ *associated to the eigenvalues* $\lambda_n=\beta-\alpha n^2\pi^2$ *(*$\alpha,\,\beta\in\mathbb{R}$*).*
For $z^*\in Z'\equiv Z=L^2(0,1)$*, we have*

$$||B^*S^*_{(\alpha,\beta)}(t)z^*||^2_{L^2(0,T;\mathbb{R})}=\sum_{n=1}^{+\infty}\int_0^T e^{2(\beta-\alpha n^2\pi^2)t}\langle z^*,\varphi_n\rangle^2\langle g,\varphi_n\rangle^2dt \tag{2.12}$$

Hence, if $g=\varphi_{n_0}$ $(n_0\geqslant 1)$*, equation (2.12) becomes*

$$||B^*S^*_{(\alpha,\beta)}(t)z^*||^2_{L^2(0,T;\mathbb{R})}=\int_0^T e^{2(\beta-\alpha n_0^2\pi^2)t}\langle z^*,\varphi_{n_0}\rangle^2dt$$

Let $A_1=M_{1,0}=\dfrac{\partial^2}{\partial x^2}$ *and* $A_2=M_{1,\beta}=\dfrac{\partial^2}{\partial x^2}+\beta I$*;* $\beta\neq 0$*.*
The corresponding semi-groups, noted $(S_1(t))_{t\geqslant 0}$ *and* $(S_2(t))_{t\geqslant 0}$*, are respectively defined by*

$$S_1(t)z=\sum_{n=1}^{+\infty}e^{-n^2\pi^2t}\langle z,\varphi_n\rangle\varphi_n$$

and

$$S_2(t)z=\sum_{n=1}^{+\infty}e^{(\beta-n^2\pi^2)t}\langle z,\varphi_n\rangle\varphi_n$$

Then for $B_1=B_2=B$*, with* $Bu=\varphi_{n_0}u$*.*

1) *If* $\beta>0$*, then for any* $z^*\in Z'$*, we have*

$$\begin{aligned}||B^*S_1^*(t)z^*||^2_{L^2(0,T;\mathbb{R})}&=\int_0^T e^{-2n_0^2\pi^2t}\langle z^*,\varphi_{n_0}\rangle^2dt\\&\leqslant\int_0^T e^{2(\beta-n_0^2\pi^2)t}\langle z^*,\varphi_{n_0}\rangle^2dt\\&=||B^*S_2^*(t)z^*||^2_{L^2(0,T;\mathbb{R})}\end{aligned}$$

consequently, the pair (A_2,B) *dominates the pair* (A_1,B) *exactly, and hence weakly.*

2) *If* $\beta<0$*, then for any* $z^*\in Z'$*,*

$$\begin{aligned}||B^*S_2^*(t)z^*||^2_{L^2(0,T;\mathbb{R})}&=\int_0^T e^{2(\beta-n_0^2\pi^2)t}\langle z^*,\varphi_{n_0}\rangle^2dt\\&\leqslant\int_0^T e^{-2n_0^2\pi^2t}\langle z^*,\varphi_{n_0}\rangle^2dt\\&=||B^*S_1^*(t)z^*||^2_{L^2(0,T;\mathbb{R})}\end{aligned}$$

Hence, the pair (A_1,B) *dominates the pair* (A_2,B) *exactly (and then weakly).*

2.3. Domination and choice of actuators

As a consequence of the previous results, we give now a characterization based on the choice of the actuators. Consider again the two following systems

$$(\mathcal{S}_1)\quad \left\{ \begin{array}{l} \dot{z}_1(t) = A_1 z_1(t) + B_1 u_1(t)\ ;\ 0 < t < T \\ \\ z_1(0) = z_{1,0} \in Z \end{array} \right. \tag{2.13}$$

$$(\mathcal{S}_2)\quad \left\{ \begin{array}{l} \dot{z}_2(t) = A_2 z_2(t) + B_2 u_2(t)\ ;\ 0 < t < T \\ \\ z_2(0) = z_{2,0} \in Z \end{array} \right. \tag{2.14}$$

Each system $(\mathcal{S}_i)$ is augmented with the output equation

$$(\mathcal{E}_i)\quad y_i(t) = C z_i(t) \text{ for } i = 1, 2$$

with the same hypothesis than in the previous section. In the case where the systems have the same dynamics $A_1 = A_2$, thus the choice of the actuators structures may characterize the domination as stated hereafter.
Assume that the system $(\mathcal{S}_1)$ is excited by an actuator (Ω_1, g_1) and that the system $(\mathcal{S}_2)$ is excited by an actuator (Ω_2, g_2) which extends the actuator (Ω_1, g_1) in the sense [29], i.e.

$$\left\{ \begin{array}{l} i)\ \ \Omega_1 \subset \Omega_2 \\ ii)\ \ g_2 = \left\{ \begin{array}{lll} g_1 & in & \Omega_1 \\ anything & in & \Omega_2 \setminus \Omega_1 \end{array} \right. \end{array} \right.$$

(Ω_2, g_2) is mathematically interpreted as two actuators $(D_i, h_i)_{1 \le i \le 2}$ defined as follows: $D_1 = \Omega_1$, $h_1 = g_1$, $D_2 = \Omega_2 \setminus \Omega_1$ and $h_2 = g_2$ on D_2.

The following result shows that practically, the more the actuator support grows the more the system is dominator.

Proposition 5.
Under the above hypothesis, the system $(\mathcal{S}_2)$ dominates the system $(\mathcal{S}_1)$ with respect to any operator C (sensors).

An other interesting result is related to the number of actuators. It can be stated as follows.

Proposition 6.
Assume that the system $(\mathcal{S}_1)$ is excited by p actuators (Ω_i, g_i) with $1 \le i \le p$ and that the system $(\mathcal{S}_2)$ is excited by q actuators (Ω_i, g_i) with $1 \le i \le q$. Then if $p \le q$, the system $(\mathcal{S}_2)$ dominates the system $(\mathcal{S}_1)$ with respect to any operator C (sensors).

This result which is more general is a consequence of the previous characterizations. It shows that, for a given system, if you add actuators then you make the system dominating.
The above results are interesting from engineering point of view because they give the user a simple way of making a system more or less dominator. The results can be easily extended, under convenient hypothesis, to the case of pointwise or boundary actuators. These results can be also extended to the domination based on sensors.

2.4. Domination based on output operators

In this section, we introduce and we study the notion of domination for observed systems (output operators) with respect to an input one. We consider first a dual problem where the control concerns the initial state, and then a general controlled system.

2.4.1. A dual problem

In this section, we examine a dual problem concerning the output operators and observed systems. We consider the system

$$\left\{ \begin{array}{l} \dot{z}(t) = Az(t); 0 < t < T \\ \\ z(0) = Bu_0 \end{array} \right. \tag{2.15}$$

The initial state z_0 depends on an input operator B and is of the form $z(0) = Bu_0$. We assume that A is a linear operator with a domain $D(A)$ dense in Z, a separable Hilbert space, and generates a strongly continuous semi-group $(S(t))_{t\geq 0}$ on the state space Z. $B \in \mathcal{L}(U, Z)$, $u_0 \in U$; U is a Hilbert space. The system (2.15) is augmented with the following output equations

$$y_1(t) = C_1 z(t) \; ; \; 0 < t < T \tag{2.16}$$

$$y_2(t) = C_2 z(t) \; ; \; 0 < t < T \tag{2.17}$$

For $i = 1, 2$; the observations are given by

$$y_i(t) = C_i S(t) B u_0 \; ; \; 0 < t < T$$

We have $y_i(.) = K_i(.)u_0$, with

$$K_i = C_i S(.) B$$

Its adjoint operator is defined by

$$K_i^* y = \int_0^T B^* S^*(t) C_i^* y(t) dt$$

Noting $B_i = C_i^*$; $i = 1, 2$; $B = C^*$ and considering the dual systems

$$(\mathcal{S}_i^*) \begin{cases} \dot{z}_i(t) &= A^* z_i(t) + B_i u_i(t) \; ; \; 0 < t < T \\ z_i(0) &= z_0 \\ y_i(t) &= C z_i(t) \end{cases}$$

and

$$(\widetilde{\mathcal{S}}_i) \begin{cases} \dot{z}(t) &= A z(t) \; ; \; 0 < t < T \\ z(0) &= B u_0 \\ y_i(t) &= C_i z(t) \end{cases}$$

we obtain the following characterization result.

Proposition 7. *$Im(K_2^*) \subset Im(K_1^*)$ (respectively $\overline{Im(K_2^*)} \subset \overline{Im(K_1^*)}$) if and only if, the controlled system $(\mathcal{S}_1^*)$ dominates $(\mathcal{S}_2^*)$ exactly (respectively weakly).*

From this general result, one can deduce analogous results and similar properties to those given in previous sections.

2.4.2. Domination of output operators

We consider the following linear distributed system

$$(\mathcal{S}) \begin{cases} \dot{z}(t) = A z(t) + B u(t) \; ; \; 0 < t < T \\ \\ z(0) = z_0 \end{cases} \tag{2.18}$$

where A generates a s.c.s.g. $(S(t))_{t\geq 0}$ on the state space Z; $B \in \mathcal{L}(U, Z)$ and $u \in L^2(0, T; U)$; U is the control space and the system (S) is augmented with the output equations

$$(\mathcal{E}_i) \;\; y_i(t) \;\; = \;\; C_i z(t) \;\; , 0 < t < T \; ; \;\; i = 1, 2.$$

where $C_i \in \mathcal{L}(Z, Y)$; $i = 1, 2$; Y is an Hilbert space. The observation with respect to operator C_i at the final time T, is given by

$$y_i(T) = C_i S(T) z_0 + C_i H u \tag{2.19}$$

We introduce hereafter the appropriate notion of domination for the considered case.

Definition 8. *We say that*

i. *C_1 dominates C_2 exactly with respect to the system (S) (or the pair (A, B)) on $[0, T]$, if $Im(C_2 H) \subset Im(C_1 H)$.*

ii. *C_1 dominates C_2 weakly with respect to the system (S) (or the pair (A, B)) on $[0, T]$, if $\overline{Im(C_2 H)} \subset \overline{Im(C_1 H)}$.*

Here also, we can deduce similar characterization results in the weak and exact cases. On the other hand, one can consider a natural question on a possible transitivity of such a domination. As it will be seen, this may be possible under convenient hypothesis. In order to examine this question, we consider without loss of generality, the linear distributed systems with the same dynamics A ($A_1 = A_2 = A$).

$$(\mathcal{S}_1)\begin{cases} \dot{z}_1(t) = Az_1(t) + B_1u_1(t)\ ;\ 0 < t < T \\ z_1(0) = z_{1,0} \end{cases}$$

$$(\mathcal{S}_2)\begin{cases} \dot{z}_2(t) = Az_2(t) + B_2u_2(t)\ ;\ 0 < t < T \\ z_2(0) = z_{2,0} \end{cases}$$

where A generates a s.c.s.g. $(S(t))_{t\geq 0}$ on the state space Z; $B_1 \in \mathcal{L}(U_1, Z)$, $B_2 \in \mathcal{L}(U_2, Z)$, $u_1 \in L^2(0,T;U_1)$, $u_2 \in L^2(0,T;U_2)$; U_1 and U_2 are two control spaces. The systems $(\mathcal{S}_1)$ and $(\mathcal{S}_2)$ are augmented with the output equations

$$\begin{array}{lcl} (\mathcal{E}_{1,i}) & : & y_{i,1}(t) = C_1z_i(t)\ ;\ i = 1,2 \\ (\mathcal{E}_{2,j}) & : & y_{j,2}(t) = C_2z_j(t)\ ;\ j = 1,2 \end{array}$$

where $C_i \in \mathcal{L}(Z,Y)$, for $i = 1,2$; Y is a Hilbert space. The observations with respect to operator C_1 at the final time T are respectively given by

$$y_{1,1}(T) = C_1S(T)z_{1,0} + C_1H_{B_1}u_1$$

$$y_{2,1}(T) = C_1S(T)z_{2,0} + C_1H_{B_2}u_2$$

By the same, the observations with respect to operator C_2 at time T are given by

$$y_{1,2}(T) = C_2S(T)z_{1,0} + C_2H_{B_1}u_1$$

$$y_{2,2}(T) = C_2S(T)z_{2,0} + C_2H_{B_2}u_2$$

We have the following result deriving from the definitions.

Proposition 9. *If the following conditions are satisfied*

i. *B_1 dominates B_2 exactly (respectively weakly) with respect to operator C_1,*

ii. *C_1 dominates C_2 exactly (respectively weakly) with respect to operator B_2,*

iii. *C_2 dominates C_1 exactly (respectively weakly) with respect to operator B_1,*

then B_1 dominates B_2 exactly (respectively weakly) with respect to operator C_2.

We study hereafter the relationship between the notions of domination and compensation.

2.5. Domination and Remediability

In this section, we study the relationship between the notions of domination and remediability. We consider without loss of generality, the following systems with $A_1 = A_2$.

$$(\mathcal{S}_1)\begin{cases} \dot{z}_1(t) = Az_1(t) + d_1(t) + B_1u_1(t), 0 < t < T \\ z_1(0) = z_{1,0} \end{cases} \tag{2.20}$$

$$(\mathcal{S}_2)\begin{cases} \dot{z}_2(t) = Az_2(t) + d_2(t) + B_2u_2(t), 0 < t < T \\ z_2(0) = z_{2,0} \end{cases} \tag{2.21}$$

where A generates a s.c.s.g. $(S(t))_{t\geq 0}$ on the state space Z; $B_1 \in \mathcal{L}(U_1, Z)$, $B_2 \in \mathcal{L}(U_2, Z)$, $u_1 \in L^2(0,T;U_1)$, $u_2 \in L^2(0,T;U_2)$, d_1 and $d_2 \in L^2(0,T;Z)$; U_1 and U_2 are two control spaces. $(\mathcal{S}_1)$ and $(\mathcal{S}_2)$ are respectively augmented with the output equations

$$(\mathcal{E}_1)\ y_1(t) = C_1z_1(t)$$

$$(\mathcal{E}_2)\ y_2(t) = C_2z_2(t)$$

The states of these systems at the final time T are respectively given by

$$\begin{aligned} z_1(T) &= S(T)z_0 + H_{B_1}u_1 + Rd_1 \\ z_2(T) &= S(T)z_0 + H_{B_2}u_2 + Rd_2 \end{aligned} \tag{2.22}$$

where the operators H_{B_i}; $i = 1, 2$ and R are defined by

$$\begin{array}{rccl} H_{B_i}: & L^2(0,T;U_i) & \longrightarrow & Z \\ & u_i & \longrightarrow & \int_0^T S(T-s)B_i u_i(s)ds \end{array} \tag{2.23}$$

$$\begin{array}{rccl} R: & L^2(0,T;Z) & \longrightarrow & Z \\ & d & \longrightarrow & \int_0^T S(T-s)d(s)ds \end{array} \tag{2.24}$$

The corresponding observations are given by

$$y_1(T) = C_1S(T)z_0 + C_1H_{B_1}u_1 + C_1Rd_1 \tag{2.25}$$

$$y_2(T) = C_2S(T)z_0 + C_2H_{B_2}u_2 + C_2Rd_2 \tag{2.26}$$

First let us recall the notion of remediability.

Definition 10.

i. *The system $(\mathcal{S}_i)$ augmented with output equation $(\mathcal{E}_i)$ (or $(\mathcal{S}_i) + (\mathcal{E}_i)$) is exactly remediable on $[0, T]$ if for any $d_i \in L^2(0, T; Z)$, there exists $u_i \in L^2(0, T; U_i)$ such that $C_iH_{B_i}u_i + C_iRd_i = 0$, or equivalently*

$$Im(C_iR) \subset Im(C_iH_{B_i}) \tag{2.27}$$

ii. *The system $(\mathcal{S}_i)$ augmented with output equation $(\mathcal{E}_i)$ (or $(\mathcal{S}_i) + (\mathcal{E}_i)$) is weakly remediable on $[0, T]$ if for any $d_i \in L^2(0, T; Z)$ and any $\epsilon > 0$, there exists $u_i \in L^2(0, T; U_i)$ such that $||C_iH_{B_i}u_i + C_iRd_i|| < \epsilon$, or equivalently*

$$Im(C_iR) \subset \overline{Im(C_iH_{B_i})} \tag{2.28}$$

Here, the question is not to examine if a system is (or not) remediable, but to study the nature of the relation between the notions of domination and remediability, respectively in the exact and weak cases. We have the following result.

Proposition 11. *If the following conditions hold*

i. *$(\mathcal{S}_1) + (\mathcal{E}_1)$ is exactly (respectively weakly) remediable.*

ii. *C_2 dominates C_1 exactly (respectively weakly) with respect to the operator B_1.*

iii. *$Im(C_2R) \subset Im(C_1R)$ (respectively $\overline{Im(C_2R)} \subset \overline{Im(C_1R)}$).*

then $(\mathcal{S}_1) + (\mathcal{S}_2)$ is exactly (respectively weakly) remediable.

We have the similar result concerning the input domination and the remediability notion.

Proposition 12. *If the following conditions are satisfied*

i. *$(\mathcal{S}_1) + (\mathcal{E}_1)$ is exactly (respectively weakly) remediable.*

ii. *B_2 dominates B_1 exactly (respectively weakly) with respect to the operator C_1.*

then $(\mathcal{S}_2) + (\mathcal{E}_1)$ is exactly (respectively weakly) remediable.

In the next section, we consider a class of parabolic systems (diffusion systems). We examine the case of a finite number of actuators, and then that where the observation is given by sensors.

3. Application to diffusion systems

3.1. Considered systems

This section is focused on the case of a class of parabolic systems and on the notions of actuators and sensors [4, 5, 7, 37, 42], i.e. on input and output operators. In what follows, we assume that $Z = L^2(\Omega)$ and, without loss of generality, we consider the analytic case where A_1 and A_2 generate respectively the s.c.s.g. $(S_1(t))_{t\geq 0}$ and $(S_2(t))_{t\geq 0}$ defined by

$$S_1(t)z = \sum_{n=1}^{+\infty} e^{\lambda_n t} \sum_{j=1}^{r_n} \langle z, \varphi_{nj} \rangle \varphi_{nj} \tag{3.1}$$

and

$$S_2(t)z = \sum_{n=1}^{+\infty} e^{\gamma_n t} \sum_{j=1}^{s_n} \langle z, \psi_{nj} \rangle \psi_{nj} \tag{3.2}$$

where $\{\varphi_{nj}, j = 1, \dots, r_n; n \geqslant 1\}$ is a complete orthonormal basis of eigenfunctions of A_1, associated to the real eigenvalues $(\lambda_n)_{n\geq 1}$ such that $\lambda_1 > \lambda_2 > \lambda_3 > \cdots$; r_n is the multiplicity of λ_n. $\{\psi_{nj}, j = 1, \dots, s_n; n \geqslant 1\}$ is a complete orthonormal basis of eigenfunctions of A_2, associated to the real eigenvalues $(\gamma_n)_{n\geq 1}$ such that $\gamma_1 > \gamma_2 > \gamma_3 > \cdots$; s_n is the multiplicity of γ_n.
In this case, the operators A_1 and A_2 are respectively defined by

$$A_1 z = \sum_{n=1}^{+\infty} \lambda_n \sum_{j=1}^{r_n} \langle z, \varphi_{nj} \rangle \varphi_{nj} \tag{3.3}$$

and

$$A_2 z = \sum_{n=1}^{+\infty} \gamma_n \sum_{j=1}^{s_n} \langle z, \psi_{nj} \rangle \psi_{nj} \tag{3.4}$$

3.2. Case of actuators

In the case where the system $(\mathcal{S}_1)$ is excited by p zone actuators $(\Omega_i, g_i)_{1\leq i\leq p}$, we have $U_1 = \mathbb{R}^p$ and

$$B_1 u(t) = \sum_{i=1}^{p} g_i u_i(t) \tag{3.5}$$

where $u = (u_1, \cdots, u_p)^{tr} \in L^2(0, T; \mathbb{R}^p)$ and $g_i \in L^2(\Omega)$; $\Omega_i = supp(g_i) \subset \Omega$. We have

$$B_1^* z = (\langle g_1, z \rangle, \cdots, \langle g_p, z \rangle)^{tr} \tag{3.6}$$

By the same, if $(\mathcal{S}_2)$ is excited by q zone actuators $(D_i, h_i)_{1\leq i\leq q}$, we have $U_2 = \mathbb{R}^q$ and

$$B_2 v(t) = \sum_{i=1}^{q} h_i v_i(t) \tag{3.7}$$

with $v = (v_1, \cdots, v_q)^{tr} \in L^2(0, T; \mathbb{R}^q)$, $h_i \in L^2(\Omega)$, $D_i = supp(h_i) \subset \Omega$ and

$$B_2^* z = (\langle h_1, z \rangle, \cdots, \langle h_q, z \rangle)^{tr} \tag{3.8}$$

As it will be seen in the next section, this leads to characterization results depending on C and the corresponding controllability matrix, and then on the observability one in the case where the observation is given by a finite number of sensors. First, let us show the following preliminary result.

Proposition 13. *We have*

$$\ker(B_1^* S_1^*(.)C^*) = \{\theta \in Y' / \forall n \in \mathbb{N}^*, (\langle C^*\theta, \varphi_{nj} \rangle)_{1\leqslant j\leqslant r_n} \in \ker(M_n)\}$$

and

$$\ker(B_2^* S_2^*(.)C^*) = \{\theta \in Y' / \forall n \in \mathbb{N}^*, (\langle C^*\theta, \psi_{nj} \rangle)_{1\leqslant j\leqslant s_n} \in \ker(Q_n)\}$$

where M_n and Q_n are the corresponding controllability matrices defined by

$$M_n = (\langle g_i, \varphi_{nj} \rangle)_{1 \leqslant i \leqslant p; 1 \leqslant j \leqslant r_n} \ \ and \ \ Q_n = (\langle h_i, \psi_{nj} \rangle)_{1 \leqslant i \leqslant q; 1 \leqslant j \leqslant s_n} \tag{3.9}$$

Proof. We have

$$B_1^* S_1^*(t) C^* \theta = \left(\sum_{n=1}^{+\infty} e^{\lambda_n t} \sum_{j=1}^{r_n} \langle C^* \theta, \varphi_{nj} \rangle \langle g_i, \varphi_{nj} \rangle \right)_{1 \leqslant i \leqslant p}$$

Therefore, $\theta \in \ker(B_1^* S_1^*(.) C^*)$ if and only if

$$\sum_{n=1}^{+\infty} e^{\lambda_n t} \sum_{j=1}^{r_n} \langle C^* \theta, \varphi_{nj} \rangle \langle g_i, \varphi_{nj} \rangle = 0; \ \forall \, i \in \{1, \cdots, p\} \, , \ \forall \, t \geq 0$$

By analyticity, this is equivalent to

$$\sum_{j=1}^{r_n} \langle C^* \theta, \varphi_{nj} \rangle \langle g_i, \varphi_{nj} \rangle = 0; \ \forall \, n \geq 1, \ \forall \, i \in \{1, \cdots, p\}$$

or

$$B_1^* S_1^*(.) C^* \theta = 0 \Longleftrightarrow v_n(\theta) \in ker(M_n), \ \forall \, n \geq 1$$

where

$$v_n(\theta) = \left(\langle C^* \theta, \varphi_{nj} \rangle \right)_{j=1, r_n}$$

The proof of the second equality of the proposition is similar. □

The following result deriving from proposition 2, gives characterizations of exact and weak domination in the case of actuators.

Proposition 14.

i. The system $(\mathcal{S}_1)$ dominates the system $(\mathcal{S}_2)$ exactly with respect to the operator C, if and only if there exists $\gamma > 0$ such that for any $\theta \in Y'$, we have

$$\begin{aligned} & \left\| \left(\sum_{n=1}^{+\infty} e^{\gamma_n t} \sum_{j=1}^{s_n} \langle C^* \theta, \psi_{nj} \rangle \langle h_i, \psi_{nj} \rangle \right)_{1 \leqslant i \leqslant q} \right\|_{L^2(0,T;\mathbb{R}^q)} \\ \leq \ \ & \gamma \left\| \left(\sum_{n=1}^{+\infty} e^{\lambda_n t} \sum_{j=1}^{r_n} \langle C^* \theta, \varphi_{nj} \rangle \langle g_i, \varphi_{nj} \rangle \right)_{1 \leqslant i \leqslant p} \right\|_{L^2(0,T;\mathbb{R}^p)} \end{aligned} \tag{3.10}$$

ii. The system $(\mathcal{S}_1)$ dominates the system $(\mathcal{S}_2)$ weakly with respect to the operator C, if and only if for any $\theta \in Y'$, we have

$$[\forall n \in \mathbb{N}^*, (\langle C^* \theta, \varphi_{nj} \rangle)_{1 \leqslant j \leqslant r_n} \in \ker M_n] \Rightarrow [\forall n \in \mathbb{N}^*, (\langle C^* \theta, \psi_{nj} \rangle)_{1 \leqslant j \leqslant s_n} \in \ker Q_n] \tag{3.11}$$

Let us note that if $A_1 = A_2$, the domination concerns the operators B_1 and B_2, and then the corresponding actuators. This leads to the following definition.

Definition 15. *If the system $(\mathcal{S}_1)$ dominates the system $(\mathcal{S}_2)$ exactly (respectively weakly) with respect to the operator C, we say that $(\Omega_i, g_i)_{1 \leqslant i \leqslant p}$ dominate $(D_i, h_i)_{1 \leqslant i \leqslant q}$ exactly (respectively weakly) with respect to C.*

In the usual case, the observation is given by sensors. This is examined in following section.

3.3. Case of sensors

Now, if the output is given by m sensors $(E_i, f_i)_{1 \leqslant i \leqslant m}$, we have

$$Cz = \begin{pmatrix} \langle z, f_1 \rangle \\ \vdots \\ \langle z, f_m \rangle \end{pmatrix} \in \mathbb{R}^m$$

and

$$C^*\theta = \sum_{i=1}^{m} \theta_i f_i \quad for \quad \theta \in \mathbb{R}^m$$

We have the following proposition.

Proposition 16. *The system $(\mathcal{S}_1)$ dominates the system $(\mathcal{S}_2)$ weakly with respect to the sensors $(E_i, f_i)_{1\leqslant i\leqslant m}$, if and only if*

$$\bigcap_{n\geq 1} \ker(M_n G_n^{tr}) \subset \bigcap_{n\geq 1} \ker(Q_n R_n^{tr}) \tag{3.12}$$

where G_n and R_n are the corresponding observability matrices defined by

$$G_n = (\langle f_i, \varphi_{nj}\rangle)_{1\leqslant i\leqslant m; 1\leqslant j\leqslant r_n} \;\; and \;\; R_n = (\langle f_i, \psi_{nj}\rangle)_{1\leqslant i\leqslant m; 1\leqslant j\leqslant s_n} \tag{3.13}$$

Proof. The system $(\mathcal{S}_1)$ dominates the system $(\mathcal{S}_2)$ weakly with respect to the sensors $(E_i, f_i)_{1\leqslant i\leqslant m}$, if and only if, for any $\theta = (\theta_k)_{1\leq k\leq m} \in \mathbb{R}^m$,

$$\forall n \in \mathbb{N}^*, (\sum_{i=1}^{m} \theta_k \langle f_k, \varphi_{nj}\rangle)_{1\leq j\leq r_n} \in \ker(M_n)$$

implies that

$$\forall n \in \mathbb{N}^*, (\sum_{i=1}^{m} \theta_k \langle f_k, \psi_{nj}\rangle)_{1\leq j\leq s_n} \in \ker(Q_n)$$

or equivalently, for any $\theta \in \mathbb{R}^m$,

$$[\forall n \geq 1, \theta \in \ker(M_n G_n^*)] \Longrightarrow [\forall n \geq 1, \theta \in \ker(Q_n G_n^*)]$$

we then have the result. □

Let us give the following remarks.

1. If $A_1 = A_2$, we have $G_n = R_n$, for $n \geq 1$. Moreover, if $(\mathcal{S}_1)$ dominates $(\mathcal{S}_2)$ with respect to the sensors $(E_i, f_i)_{1\leq i\leq m}$, then $(\mathcal{S}_1)$ dominates $(\mathcal{S}_2)$ with respect to $(E_i, f_i)_{1\leq i\leq q}$ for $1 \leq q \leq m$.
2. One actuator may dominates p actuators ($p > 1$), with respect to an output operator C (sensors).
3. In the case of one actuator and one sensor, i.e. for $p = q = 1$ and $m = 1$, we have

   $$M_n = (\langle g, \varphi_{n1}\rangle, \cdots, \langle g, \varphi_{nr_n}\rangle), Q_n = (\langle h, \psi_{n1}\rangle, \cdots, \langle h, \psi_{ns_n}\rangle)$$

   and

   $$G_n^{tr} = \begin{pmatrix} \langle f, \varphi_{n1}\rangle \\ \vdots \\ \langle f, \varphi_{nr_n}\rangle \end{pmatrix} \quad , \quad R_n^{tr} = \begin{pmatrix} \langle f, \psi_{n1}\rangle \\ \vdots \\ \langle f, \psi_{ns_n}\rangle \end{pmatrix}$$

   Then

   $$\begin{aligned} M_n G_n^{tr} &= (\sum_{j=1}^{r_n} < g, \varphi_{nj}\rangle \langle f, \varphi_{nj}\rangle) \\ Q_n R_n^{tr} &= (\sum_{j=1}^{s_n} < h, \psi_{nj}\rangle \langle f, \psi_{nj}\rangle) \end{aligned} \tag{3.14}$$

4. In the case of a finite number of sensors, the exact and weak domination are equivalent.

3.4. Applications

To illustrate the previous results and other specific situations, we consider without loss of generality, a class of diffusion systems described by the following parabolic equation.

$$(\mathcal{S})\begin{cases} \dfrac{\partial z(x,t)}{\partial t} = \Delta z(x,t) + g(x)u(t) & \Omega\times]0,T[\\ z(x,0) = 0 & \Omega \\ z(\xi,t) = 0 & \partial\Omega\times]0,T[\end{cases} \tag{3.15}$$

where Ω is a bounded subset of $\mathbb{R}^n$ with a sufficiently regular boundary $\partial\Omega = \Gamma$; $Z = L^2(\Omega)$ and $Az = \Delta z$ for $z \in D(A) = H^2(\Omega) \cap H_0^1(\Omega)$. (S) is augmented with the output equation

$$(\mathcal{E})\ \ y(t) = Cz(t),\ \ 0 < t < T \tag{3.16}$$

We explore hereafter respectively the case of one and two space dimension.

3.4.1. One dimension case

In this section, we consider the systems $(\mathcal{S}_1)$ and $(\mathcal{S}_2)$ described by the following one dimension equations, with $\Omega =]0,a[$ and $A_1 = A_2 = \Delta$.

$$(\mathcal{S}_1)\begin{cases} \dfrac{\partial z_1(x,t)}{\partial t} = \dfrac{\partial^2 z_1(x,t)}{\partial x^2} + g(x)u_1(t) &]0,a[\times]0,T[\\ z_1(0,t) = z_1(a,t) = 0 &]0,T[\\ z_1(x,0) = 0 &]0,a[\end{cases} \tag{3.17}$$

$$(\mathcal{S}_2)\begin{cases} \dfrac{\partial z_2(x,t)}{\partial t} = \dfrac{\partial^2 z_2(x,t)}{\partial x^2} + h(x)u_2(t) &]0,a[\times]0,T[\\ z_2(0,t) = z_2(a,t) = 0 &]0,T[\\ z_2(x,0) = 0 &]0,a[\end{cases} \tag{3.18}$$

$A = \Delta$ admits a complete orthonormal system of eigenfunctions $(\varphi_n)_{n\in\mathbb{N}^*}$ associated to the eigenvalues $\lambda_n = -\dfrac{n^2\pi^2}{a^2}$ with $\varphi_n(x) = \sqrt{\dfrac{2}{a}}\sin(\dfrac{n\pi x}{a})$.
Each system $(\mathcal{S}_i)$ is augmented with the output equation corresponding to a sensor (D, f),

$$(\mathcal{E}_i)\ y_i(t) = \langle f, z_i(t)\rangle_{L^2(D)}\ ;\ 0\ < t\ <\ T \tag{3.19}$$

According to proposition 16, (Ω, g) dominates (Ω, h) with respect to the sensor (D, f), if and only if,

$$[\forall n \in \mathbb{N}^*, \langle g, \varphi_n\rangle\langle f, \varphi_n\rangle = 0] \Longrightarrow [\forall n \in \mathbb{N}^*, \langle h, \varphi_n\rangle\langle f, \varphi_n\rangle = 0] \tag{3.20}$$

Let $m, n \in \mathbb{N}^*$ such that $m \neq n$. We suppose that $(\mathcal{S}_1)$ and $(\mathcal{S}_2)$ are respectively excited by the actuators (Ω, φ_n) and (Ω, φ_m)., i.e. $g = \varphi_n$ and $h = \varphi_m$.

Then

- (Ω, g) dominates (Ω, h) with respect to the sensor (Ω, φ_n) and
- (Ω, h) dominates (Ω, g) with respect to the sensor (Ω, φ_m).

Let us also note that in the one dimension case, any operators B_1 and B_2 are comparable. This is not always possible in the two-dimension case which will be examined in the next section.

3.4.2. Two dimension case

Now, we consider the case where $\Omega =]0,1[\times]0,1[$ and the systems described by the following equations

$$(\mathcal{S}_1)\begin{cases} \dfrac{\partial z_1(x,y,t)}{\partial t} = \Delta z_1(x,y,t) + g_1(x,y)u_1(t) + g_2(x,y)u_2(t) & \Omega\times]0,T[\\ z_1(x,y,t) = 0 & \Gamma\times]0,T[\\ z_1(x,y,0) = 0 & \Omega \end{cases}$$

$$(\mathcal{S}_2)\begin{cases} \dfrac{\partial z_2(x,y,t)}{\partial t} = \Delta z_2(x,y,t) + h_1(x,y)v_1(t) + h_2(x,y)v_2(t) & \Omega\times]0,T[\\ z_2(x,y,t) = 0 & \Gamma\times]0,T[\\ z_2(x,y,0) = 0 & \Omega \end{cases}$$

Here, we have $Z = L^2(\Omega)$ and $Az = \Delta z = \dfrac{\partial z}{\partial x^2} + \dfrac{\partial z}{\partial y^2}$, for $z \in D(A) = H^2(\Omega) \cap H_0^1(\Omega)$. A admits a complete orthonormal system of eigenfunctions $(\varphi_{m,n})_{m,n\in\mathbb{N}^*}$ associated to the eigenvalues $(\lambda_{m,n})_{m,n\in\mathbb{N}^*}$ defined by

$$\begin{cases} \lambda_{m,n} = -(m^2+n^2)\pi^2 \\ \varphi_{m,n}(x,y) = 2\sin(m\pi x)\sin(n\pi y) \end{cases} \tag{3.21}$$

$(\mathcal{S}_1)$ and $(\mathcal{S}_2)$ are respectively augmented with the output equations

$$(\mathcal{E}_1)\ y_1(t) = (\langle f_1, z_1(t)\rangle_{L^2(D_1)}, \langle f_2, z_1(t)\rangle_{L^2(D_2)}),\ 0 < t < T \tag{3.22}$$

and

$$(\mathcal{E}_2)\ y_2(t) = (\langle f_1, z_2(t)\rangle_{L^2(D_1)}, \langle f_2, z_2(t)\rangle_{L^2(D_2)}),\ 0 < t < T \tag{3.23}$$

Let us first note that: $200 = 14^2 + 2^2 = 10^2 + 10^2$, then -200 is a double eigenvalue, corresponding to the eigenfunctions $\varphi_{10,10}$ and $\varphi_{2,14}$.

By the same, $250 = 15^2 + 5^2 = 13^2 + 9^2$, then -250 is also a double eigenvalue, corresponding to the eigenfunctions $\varphi_{5,15}$ and $\varphi_{9,13}$.

The given examples hereafter show the following situations :
- An actuator may dominates another one with respect to a sensor.
- None of the systems does not dominate the other.

Example 17. *In the case where $g_1 = \varphi_{10,10}$, $g_2 = 0$, $h_1 = 0, h_2 = \varphi_{5,15}$, $f_1 = \varphi_{10,10}$ and $f_2 = \varphi_{2,14}$, we have*

$$\bigcap_{n\geq 1} \ker(M_n G_n^{tr}) = \mathbb{R}(0,1) \text{ and } \bigcap_{n\geq 1} \ker(Q_n G_n^{tr}) = \{0\} \tag{3.24}$$

where $\mathbb{R}(0,1)$ denotes the y-axis. Therefore the system $(\mathcal{S}_2)$ dominates the system $(\mathcal{S}_1)$ with respect to the corresponding output operator C.
On the other hand, for $g_1 = \varphi_{10,10}$, $g_2 = 0$, $h_1 = 0$, $h_2 = \varphi_{5,15}$ and $f_1 = \varphi_{5,15}$, $f_2 = \varphi_{9,13}$, we have

$$\bigcap_{n\geq 1} \ker(M_n G_n^{tr}) = \{0\} \text{ and } \bigcap_{n\geq 1} \ker(Q_n G_n^{tr}) = \mathbb{R}(1,0) \tag{3.25}$$

where $\mathbb{R}(1,0)$ denotes the x-axis. Then the system $(\mathcal{S}_1)$ dominates the system $(\mathcal{S}_2)$ with respect to the corresponding output operator C.

Example 18. *Now, for $g_1 = \varphi_{10,10}$, $g_2 = 0$, $h_1 = 0, h_2 = \varphi_{2,14}$, $f_1 = \varphi_{10,10}$ and $f_2 = \varphi_{2,14}$, we have*

$$\bigcap_{n\geq 1} \ker(M_n G_n^{tr}) = \mathbb{R}(0,1) \text{ and } \bigcap_{n\geq 1} \ker(Q_n G_n^{tr}) = \mathbb{R}(1,0) \tag{3.26}$$

Then none of the operators B_1 and B_2 does not dominates the other.

4. Domination in hyperbolic systems

4.1. Introduction and considered systems

In this section, we examine the problem of domination for a general class of hyperbolic systems. Let us first consider the particular situation of a wave equation

$$\left\{\begin{array}{lcll} \dfrac{\partial^2 x}{\partial t^2}(\xi,t) & = & \Delta x(\xi,t)+Bu(t) & \Omega\times]0,T[\\ x(\xi,0) & = & \dfrac{\partial x}{\partial t}(\xi,0)=0 & \Omega \\ x(\eta,t) & = & 0 & \partial\Omega\times]0,T[\end{array}\right. \tag{4.1}$$

where Ω is an open, bounded and sufficiently regular subset of $\mathbb{R}^n$. $B\in\mathcal{L}(U,L^2(\Omega))$, $u\in L^2(0,T;U)$; U is a control space, a Hilbert space; Δ is the Laplacian operator, T is large enough. The system (4.1) is augmented with the following output equation

$$y(t)=\begin{pmatrix} C_1x(.,t) \\ \\ C_2\dfrac{\partial x}{\partial t}(.,t)\end{pmatrix} \tag{4.2}$$

where $C_1\in\mathcal{L}(L^2(\Omega),Y_1)$, $C_2\in\mathcal{L}(L^2(\Omega),Y_2)$, Y_1 and Y_2 are observation spaces, Hilbert spaces. Let A be the operator defined by $A\psi=\Delta\psi$ for $\psi\in D(A)=H^2(\Omega)\cap H_0^1(\Omega)$, and $z=\begin{pmatrix} x \\ \frac{\partial x}{\partial t}\end{pmatrix}\in L^2(0,T;Z)$ with $Z=H_0^1(\Omega)\times L^2(\Omega)$. The system (4.1) is equivalent to

$$(\mathcal{S})\ \left\{\begin{array}{l} \dot z(t)=\mathcal{A}z(t)+\mathcal{B}u(t);0<t<T \\ z(0)=0\end{array}\right. \tag{4.3}$$

and the output equation can be written as follows

$$y(t)=Cz(t) \tag{4.4}$$

where $\mathcal{A}$ is the operator defined by

$$\mathcal{A}=\begin{pmatrix} 0 & I \\ A & 0\end{pmatrix} \tag{4.5}$$

with $D(\mathcal{A})=\ D(A)\times H_0^1(\Omega)$. The adjoint operator $\mathcal{A}^*$ of $\mathcal{A}$ is given by $\mathcal{A}^*=-\mathcal{A}$. The operator $\mathcal{B}$ is defined by $\mathcal{B}=\begin{pmatrix} 0 \\ B\end{pmatrix}$, its adjoint is defined by $\mathcal{B}^*=\begin{pmatrix} 0 & B^*\end{pmatrix}$ and $C\in\mathcal{L}(Z,Y)$ is defined by

$$C=\begin{pmatrix} C_1 & 0 \\ 0 & C_2\end{pmatrix} \text{ and } C^*=\begin{pmatrix} C_1^* & 0 \\ 0 & C_2^*\end{pmatrix} \tag{4.6}$$

where $Y=Y_1\times Y_2$. The operator $\mathcal{A}$ is linear, closed with a dense domain in the state space Z, and generates on Z a strongly continuous semi-group (s.c.s.g) $(S(t))_{t\geq 0}$ defined by

$$S(t)\begin{pmatrix} z_1 \\ z_2\end{pmatrix}=\begin{pmatrix} \displaystyle\sum_{n=1}^{+\infty}\sum_{j=1}^{r_n}[\langle z_1,\varphi_{nj}\rangle_\Omega\cos(\sqrt{-\lambda_n}t) \\ \\ \displaystyle+\frac{1}{\sqrt{-\lambda_n}}\langle z_2,\varphi_{nj}\rangle_\Omega\sin(\sqrt{-\lambda_n}t)]\varphi_{nj} \\ \\ \displaystyle\sum_{n=1}^{+\infty}\sum_{j=1}^{r_n}[-\sqrt{-\lambda_n}\langle z_1,\varphi_{nj}\rangle_\Omega\sin(\sqrt{-\lambda_n}t) \\ \\ +\langle z_2,\varphi_{nj}\rangle_\Omega\cos(\sqrt{-\lambda_n}t)]\varphi_{nj}\end{pmatrix} \tag{4.7}$$

where $\langle .,.\rangle_\Omega$ is the inner product in $L^2(\Omega)$ and $(\varphi_{nj})_{\substack{j=1,r_n \\ n\geq 1}}$ is a complete orthonormal system of eigenfunctions of A, associated to the eigenvalues $(\lambda_n)_{n\geq 1}$ such that $0>\lambda_1>\lambda_2>\lambda_3>\cdots$; r_n is the multiplicity of λ_n and $\displaystyle\sum_n\frac{1}{\mid\lambda_n\mid}<+\infty$. The adjoint semi-group is defined by $S^*(t)=-S(-t);\forall t\geq 0$. Z is a Hilbert space with the inner product $\langle z,z'\rangle_Z=\langle(-A)^{\frac12}z_1,(-A)^{\frac12}z_1'\rangle_\Omega+\langle z_2,z_2'\rangle_\Omega$ for $z=(z_1,z_2)$ and $z'=(z_1',z_2')\in Z$.

In this section, we consider a more general class of hyperbolic distributed systems described by a state equation as follows

$$(\mathcal{S}) \left\{ \begin{array}{l} \dot{z}(t) = \mathcal{A}z(t) + \mathcal{B}u(t); 0 < t < T \\ z(0) = 0 \end{array} \right. \tag{4.8}$$

where $\mathcal{A}$ is defined by (4.5), $\mathcal{B} = \begin{pmatrix} B_1 & B_2 \\ B_3 & B_4 \end{pmatrix} \in \mathcal{L}(U, Z)$, with $U = U_1 \times U_2$; U_1 and U_2 are two control spaces, $u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \in U$ and $z = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} \in Z$. The adjoint of $\mathcal{B}$ is defined by $\mathcal{B}^* = \begin{pmatrix} B_1^* & B_3^* \\ B_2^* & B_4^* \end{pmatrix}$. The system (4.8) is augmented with the output equation (4.4). Let $\mathcal{K}_C$ be the operator defined by

$$\mathcal{K}_C : \quad u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \in L^2(0, T; U) \quad \longrightarrow \quad \int_0^T CS(T-s)\mathcal{B}u(s)ds \ \in Y \tag{4.9}$$

In the considered case , we have

$$\mathcal{K}_C u = \begin{pmatrix} \displaystyle\sum_{n=1}^{+\infty}\sum_{j=1}^{r_n}\int_0^T [\langle B_1u_1(s) + B_2u_2(s), \varphi_{nj}\rangle_\Omega \cos(\sqrt{-\lambda_n}s) \\ \displaystyle +\frac{1}{\sqrt{-\lambda_n}}\langle B_3u_1(s) + B_4u_2(s), \varphi_{nj}\rangle_\Omega \sin(\sqrt{-\lambda_n}t)]dsC_1\varphi_{nj} \\ \displaystyle\sum_{n=1}^{+\infty}\sum_{j=1}^{r_n}\int_0^T [-\sqrt{-\lambda_n}\langle B_1u_1(s) + B_2u_2(s), \varphi_{nj}\rangle_\Omega \sin(\sqrt{-\lambda_n}t) \\ +\langle B_3u_1(s) + B_4u_2(s), \varphi_{nj}\rangle_\Omega \cos(\sqrt{-\lambda_n}t)]dsC_2\varphi_{nj} \end{pmatrix}$$

Note that the system (4.8) is more general than (4.3) which may be obtained by considering $B_1 = B_2 = B_3 = 0$, then $\mathcal{B} = \begin{pmatrix} 0 & 0 \\ 0 & B \end{pmatrix} \equiv \begin{pmatrix} 0 \\ B \end{pmatrix}$.

4.2. Considered systems and definitions

We consider the following linear distributed systems

$$(\mathcal{S}) \left\{ \begin{array}{l} \dot{z}(t) = \mathcal{A}z(t) + \mathcal{B}u(t); 0 < t < T \\ z(0) = z_0 \end{array} \right. \tag{4.10}$$

$$(\widetilde{\mathcal{S}}) \left\{ \begin{array}{l} \dot{\widetilde{z}}(t) = \widetilde{\mathcal{A}}\widetilde{z}(t) + \widetilde{\mathcal{B}}\widetilde{u}(t); 0 < t < T \\ \widetilde{z}(0) = \widetilde{z}_0 \end{array} \right. \tag{4.11}$$

where $\mathcal{A}$ and $\widetilde{\mathcal{A}}$ generate s.c.s.g $(S(t))_{t\geq 0}$ and $(\widetilde{S}(t))_{t\geq 0}$ respectively; $\mathcal{B} \in \mathcal{L}(U, Z)$, $\widetilde{\mathcal{B}} \in \mathcal{L}(\widetilde{U}, Z)$, $u \in L^2(0, T; U)$ and $\widetilde{u} \in L^2(0, T; \widetilde{U})$ with $U = U_1 \times U_2$ and $\widetilde{U} = \widetilde{U}_1 \times \widetilde{U}_2$; U_1, $U_2, \widetilde{U}_1$ and $\widetilde{U}_2$ are control spaces and $Z = H_0^1(\Omega) \times L^2(\Omega)$. The systems $(\mathcal{S})$ and $(\widetilde{\mathcal{S}})$ are augmented with the output equations

$$(\mathcal{E}) \ \ y(t) = Cz(t) \tag{4.12}$$

$$(\widetilde{\mathcal{E}}) \ \ \widetilde{y}(t) = C\widetilde{z}(t) \tag{4.13}$$

where $\mathcal{C} \in \mathcal{L}(Z, Y)$. The observations, are respectively given by

$$y(T) = CS(T)z_0 + \mathcal{K}_C u \ \text{ and } \ \widetilde{y}(T) = C\widetilde{S}(T)z_0 + \widetilde{\mathcal{K}}_C\widetilde{u} \tag{4.14}$$

$\mathcal{K}_C$ and $\widetilde{\mathcal{K}}_C$ are the operators defined as follows

$$\mathcal{K}_C : \quad u = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \in L^2(0, T; U) \quad \longrightarrow \quad \int_0^T CS(T-s)\mathcal{B}u(s)ds \ \in Y \tag{4.15}$$

$$\widetilde{\mathcal{K}}_C : \quad \widetilde{u} = \begin{pmatrix} \widetilde{u}_1 \\ \widetilde{u}_2 \end{pmatrix} \in L^2(0, T; \widetilde{U}) \quad \longrightarrow \quad \int_0^T C\widetilde{S}(T-s)\widetilde{\mathcal{B}}\widetilde{u}(s)ds \in Y \tag{4.16}$$

We have the analogous definitions.

Definition 19. *We say that*

i. The system $(\mathcal{S})$ *dominates the system* $(\widetilde{\mathcal{S}})$ *(or the pair* $(\mathcal{A},\mathcal{B})$ *dominates* $(\widetilde{\mathcal{A}},\widetilde{\mathcal{B}})$*) exactly on* $[0,T]$*, with respect to the operator* C*, if* $Im(\widetilde{\mathcal{K}}_C)\subset Im(\mathcal{K}_C)$.

ii. The system $(\mathcal{S})$ *dominates the system* $(\widetilde{\mathcal{S}})$ *(or the pair* $(\mathcal{A},\mathcal{B})$ *dominates* $(\widetilde{\mathcal{A}},\widetilde{\mathcal{B}})$*) weakly on* $[0,T]$*, with respect to the operator* C*, if* $\overline{Im(\widetilde{\mathcal{K}}_C)}\subset \overline{Im(\mathcal{K}_C)}$.

We note respectively $(\mathcal{A},\mathcal{B})\underset{C}{\leq}(\widetilde{\mathcal{A}},\widetilde{\mathcal{B}})$ *and* $(\mathcal{A},\mathcal{B})\underset{C}{\lesssim}(\widetilde{\mathcal{A}},\widetilde{\mathcal{B}})$.

Let us note that the properties and remarks given in section 2, remain practically the same in the hyperbolic case. We give hereafter characterization results concerning the exact and weak domination.

4.3. Characterizations

The following result gives a characterization of the exact domination with respect to the output operator C. We assume that $Z=H_0^1(\Omega)\times L^2(\Omega)$ and that $\mathcal{A}$ and $\widetilde{\mathcal{A}}$ generate respectively the s.c.s.g. defined by

$$S(t)\begin{pmatrix} z_1\\ z_2\end{pmatrix}=\begin{pmatrix} \displaystyle\sum_{n=1}^{+\infty}\sum_{j=1}^{r_n}[\langle z_1,\varphi_{nj}\rangle_\Omega\cos(\sqrt{-\lambda_n}t) \\ \displaystyle+\frac{1}{\sqrt{-\lambda_n}}\langle z_2,\varphi_{nj}\rangle_\Omega\sin(\sqrt{-\lambda_n}t)]\varphi_{nj} \\ \displaystyle\sum_{n=1}^{+\infty}\sum_{j=1}^{r_n}[-\sqrt{-\lambda_n}\langle z_1,\varphi_{nj}\rangle_\Omega\sin(\sqrt{-\lambda_n}t) \\ +\langle z_2,\varphi_{nj}\rangle_\Omega\cos(\sqrt{-\lambda_n}t)]\varphi_{nj}\end{pmatrix} \tag{4.17}$$

$$\widetilde{S}(t)\begin{pmatrix} z_1\\ z_2\end{pmatrix}=\begin{pmatrix} \displaystyle\sum_{n=1}^{+\infty}\sum_{j=1}^{s_n}[\langle z_1,\psi_{nj}\rangle_\Omega\cos(\sqrt{-\gamma_n}t) \\ \displaystyle+\frac{1}{\sqrt{-\gamma_n}}\langle z_2,\psi_{nj}\rangle_\Omega\sin(\sqrt{-\gamma_n}t)]\psi_{nj} \\ \displaystyle\sum_{n=1}^{+\infty}\sum_{j=1}^{s_n}[-\sqrt{-\gamma_n}\langle z_1,\psi_{nj}\rangle_\Omega\sin(\sqrt{-\gamma_n}t) \\ +\langle z_2,\psi_{nj}\rangle_\Omega\cos(\sqrt{-\gamma_n}t)]\psi_{nj}\end{pmatrix} \tag{4.18}$$

where $\{\varphi_{nj}, j=1,\dots,r_n; n\geq 1\}$ (respectively $\{\psi_{nj}, j=1,\dots,s_n; n\geq 1\}$) is a complete orthonormal system of eigenfunctions of A (respectively $\widetilde{A}$), associated to the real eigenvalues $(\lambda_n)_{n\geq 1}$ such that $0>\lambda_1>\lambda_2>\lambda_3>\cdots$, where r_n is the multiplicity of λ_n (respectively $(\gamma_n)_{n\geq1}$, with $0>\gamma_1>\gamma_2>\gamma_3>\cdots$ where s_n is the multiplicity of γ_n) with $\displaystyle\sum_n\frac{1}{\mid\lambda_n\mid}<+\infty$ and $\displaystyle\sum_n\frac{1}{\mid\gamma_n\mid}<+\infty$.

We give hereafter, a characterization of the exact domination.

Proposition 20. *The following properties are equivalent*

i. The system $(\mathcal{S})$ *dominates* $(\widetilde{\mathcal{S}})$ *exactly on* $[0,T]$*, with respect to* C.

ii. For any $\widetilde{u}\in L^2(0,T;\widetilde{U})$*, there exists* $u\in L^2(0,T;U)$ *such that*

$$\mathcal{K}_Cu+\widetilde{\mathcal{K}}_C\widetilde{u}=0 \tag{4.19}$$

iii. There exists $\alpha > 0$ such that for any $\theta \equiv (\theta_1, \theta_2) \in Y'$, we have

$$\left\| \begin{pmatrix} \displaystyle\sum_{n\geq 1}\sum_{j=1}^{s_n}[(\langle C_1^*\theta_1, \psi_{nj}\rangle_\Omega \cos(\sqrt{-\gamma_n}t) - \frac{1}{\sqrt{-\gamma_n}}\langle C_2^*\theta_2, \psi_{nj}\rangle_\Omega \sin(\sqrt{-\gamma_n}t))\widetilde{B}_1^*\psi_{nj} \\ +(\sqrt{-\gamma_n}\langle C_1^*\theta_1, \psi_{nj}\rangle_\Omega \sin(\sqrt{-\gamma_n}t) + \langle C_2^*\theta_2, \psi_{nj}\rangle_\Omega \cos(\sqrt{-\gamma_n}t))\widetilde{B}_3^*\psi_{nj}] \\ \displaystyle\sum_{n\geq 1}\sum_{j=1}^{s_n}[(\langle C_1^*\theta_1, \psi_{nj}\rangle_\Omega \cos(\sqrt{-\gamma_n}t) - \frac{1}{\sqrt{-\gamma_n}}\langle C_2^*\theta_2, \psi_{nj}\rangle_\Omega \sin(\sqrt{-\gamma_n}t))\widetilde{B}_2^*\psi_{nj} \\ +(\sqrt{-\gamma_n}\langle C_1^*\theta_1, \psi_{nj}\rangle_\Omega \sin(\sqrt{-\gamma_n}t) + \langle C_2^*\theta_2, \psi_{nj}\rangle_\Omega \cos(\sqrt{-\gamma_n}t))\widetilde{B}_4^*\psi_{nj}] \end{pmatrix} \right\|$$

$$\leq \alpha \left\| \begin{pmatrix} \displaystyle\sum_{n\geq 1}\sum_{j=1}^{r_n}[(\langle C_1^*\theta_1, \varphi_{nj}\rangle_\Omega \cos(\sqrt{-\lambda_n}t) - \frac{1}{\sqrt{-\gamma_n}}\langle C_2^*\theta_2, \varphi_{nj}\rangle_\Omega \sin(\sqrt{-\lambda_n}t))B_1^*\varphi_{nj} \\ +(\sqrt{-\lambda_n}\langle C_1^*\theta_1, \varphi_{nj}\rangle_\Omega \sin(\sqrt{-\lambda_n}t) + \langle C_2^*\theta_2, \varphi_{nj}\rangle_\Omega \cos(\sqrt{-\lambda_n}t))B_3^*\varphi_{nj}] \\ \displaystyle\sum_{n\geq 1}\sum_{j=1}^{r_n}[(\langle C_1^*\theta_1, \varphi_{nj}\rangle_\Omega \cos(\sqrt{-\lambda_n}t) - \frac{1}{\sqrt{-\lambda_n}}\langle C_2^*\theta_2, \varphi_{nj}\rangle_\Omega \sin(\sqrt{-\lambda_n}t))B_2^*\varphi_{nj} \\ +(\sqrt{-\lambda_n}\langle C_1^*\theta_1, \psi_{nj}\rangle_\Omega \sin(\sqrt{-\lambda_n}t) + \langle C_2^*\theta_2, \varphi_{nj}\rangle_\Omega \cos(\sqrt{-\lambda_n}t))B_4^*\varphi_{nj}] \end{pmatrix} \right\|$$

Let us note that
- if $\mathcal{A} = \widetilde{\mathcal{A}}$, then $\gamma_n = \lambda_n$; $r_n = s_n$ and $\psi_{nj} = \varphi_{nj}$ for $n \geq 1$ and $1 \leq j \leq r_n$.
- in the case where $\widetilde{A} = aA + bI$, with a convenient choice of the reals a and b, we have: $\gamma_n = a\lambda_n + b$; $r_n = s_n$ and $\psi_{nj} = \varphi_{nj}$ for $n \geq 1$ and $1 \leq j \leq r_n$.

Concerning the weak domination, we have the following proposition.

Proposition 21. *The system $(\mathcal{S})$ dominates $(\widetilde{\mathcal{S}})$ weakly on $[0, T]$, with respect to C, if and only if*

$$\ker[\mathcal{B}^* S^*(T-.)C^*] \subset \ker[\widetilde{\mathcal{B}}^* \widetilde{S}^*(T-.)C^*] \tag{4.20}$$

In the next section, we examine the case of a finite number of actuators, and then the case where the observation is given by sensors.

4.4. Case of actuators and sensors

This section is focused on the notions of actuators and sensors [7, 5]. We consider without loss of generality, the case where

$$\mathcal{B} = \begin{pmatrix} 0 \\ B \end{pmatrix} \text{ and } \widetilde{\mathcal{B}} = \begin{pmatrix} 0 \\ \widetilde{B} \end{pmatrix}$$

4.4.1. Case of actuators

If the system $(\mathcal{S})$ is excited by p zone actuators $(\Omega_i, g_i)_{1\leq i\leq p}$, we have $U = \mathbb{R}^p$ and $Bu(t) = \sum_{i=1}^{p} g_i u_i(t)$, i.e. $\mathcal{B}u(t) = (0, \sum_{i=1}^{p} g_i u_i(t))^{tr}$, where $u = (u_1, \cdots, u_p)^{tr} \in L^2(0, T; \mathbb{R}^p)$ and $g_i \in L^2(\Omega_i)$; $\Omega_i = supp(g_i) \subset \Omega$. We have

$$B^* z = (\langle g_1, z_2\rangle, \cdots, \langle g_p, z_2\rangle)^{tr} \;\; , \;\; \mathcal{B}^* = (0, B^*)$$

By the same, if $(\widetilde{\mathcal{S}})$ is excited by q zone actuators $(\widetilde{\Omega}_i, \widetilde{g}_i)_{1\leq i\leq q}$, we have $\widetilde{U} = \mathbb{R}^q$ and $\widetilde{B}\widetilde{u}(t) = \sum_{i=1}^{q} \widetilde{g}_i \widetilde{u}_i(t)$ with $\widetilde{u} = (\widetilde{u}_1, \cdots, \widetilde{u}_q)^{tr} \in L^2(0, T; \mathbb{R}^q)$, $\widetilde{g}_i \in L^2(\widetilde{\Omega}_i)$, $\widetilde{B}^* z = (\langle \widetilde{g}_1, z_2\rangle, \cdots, \langle \widetilde{g}_q, z_2\rangle)^{tr}$ and $\widetilde{\mathcal{B}} = (0, \widetilde{B})^{tr}$.

We give hereafter a preliminary characterization result. It concerns a necessary and sufficient condition for weak domination in the particular case where the operator $C = I$. But let us first give the following lemma deriving from Fourier transform.

Lemma 22. *If* $\sum_n a_n \cos(\sqrt{-\lambda_n}t) = 0$ *and* $\sum_n b_n \sin(\sqrt{-\lambda_n}t) = 0, \quad \forall t > 0$, *with* $\sum_n a_n$ *and* $\sum_n b_n$ *absolutely convergent, then* $a_n = b_n = 0, \ \forall n \geq 1$.

Proposition 23. *The system* $(\mathcal{S})$ *dominates* $(\widetilde{\mathcal{S}})$ *weakly on any time interval* $[0, T]$, *if and only if*

$$\bigcap_{n\geq 1} \ker[M_n f_n] \subset \bigcap_{n\geq 1} \ker[\widetilde{M}_n \widetilde{f}_n] \tag{4.21}$$

$$\text{where } M_n = \left(\ \langle g_i, \varphi_{nj} \rangle \ \right)_{1\leq i\leq p; 1\leq j\leq r_n} \quad ; \quad \widetilde{M}_n = \left(\ \langle \widetilde{g}_i, \psi_{nj} \rangle \ \right)_{1\leq i\leq q; 1\leq j\leq s_n}$$

are the controllability matrices corresponding to the actuators $(\Omega_i, g_i)_{1\leq i\leq p}$ *and* $(\widetilde{\Omega}_i, \widetilde{g}_i)_{1\leq i\leq q}$ *respectively, and*

$$f_n(w) = \begin{pmatrix} \langle w, \varphi_{n1} \rangle \\ \vdots \\ \langle w, \varphi_{nr_n} \rangle \end{pmatrix} \quad ; \quad \widetilde{f}_n(w) = \begin{pmatrix} \langle w, \psi_{n1} \rangle \\ \vdots \\ \langle w, \psi_{ns_n} \rangle \end{pmatrix}$$

Proof. We assume that $\ker(\mathcal{B}^* S^*(.)) \subset \ker(\widetilde{\mathcal{B}}^* \widetilde{S}^*(.))$. We have

$$\mathcal{B}^* S^*(t) \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} \sum_{n\geq 1} [(\sum_{j=1}^{r_n} \sqrt{-\lambda_n} \langle z_1, \varphi_{nj} \rangle \langle g_i, \varphi_{nj} \rangle_{1\leq i\leq p}) \sin(\sqrt{-\lambda_n}t) \\ +(\sum_{j=1}^{r_n} \langle z_2, \varphi_{nj} \rangle \langle g_i, \varphi_{nj} \rangle_{1\leq i\leq p}) \cos(\sqrt{-\lambda_n}t)] \end{pmatrix}_{1\leq i\leq p}$$

therefore, $(z_1, z_2)^{tr} \in \ker(\mathcal{B}^* S^*(.))$ if and only if, for $1 \leq i \leq p$ and $t > 0$

$$\forall n \geq 1 \begin{cases} \sum_{j=1}^{r_n} \sqrt{-\lambda_n} \langle z_1, \varphi_{nj} \rangle \langle g_i, \varphi_{nj} \rangle_{1\leq i\leq p} \sin(\sqrt{-\lambda_n}t) = 0 \\ \sum_{j=1}^{r_n} \langle z_2, \varphi_{nj} \rangle \langle g_i, \varphi_{nj} \rangle \cos(\sqrt{-\lambda_n}t) = 0 \end{cases}$$

Using Lemma 22, we have for $n \geq 1$ and $1 \leq i \leq p$

$$\sum_{j=1}^{r_n} \langle z_1, \varphi_{nj} \rangle \langle g_i, \varphi_{nj} \rangle = 0 \quad \text{and} \quad \sum_{j=1}^{r_n} \langle z_2, \varphi_{nj} \rangle \langle g_i, \varphi_{nj} \rangle = 0$$

i.e. for any $n \geq 1$, we have $f_n(z_1) \in \ker(M_n)$ and $f_n(z_2) \in \ker(M_n)$. Since $\ker(\mathcal{B}^* S^*(.)) \subset \ker(\widetilde{\mathcal{B}}^* \widetilde{S}^*(.))$, we deduce by the same that $\forall n \geq 1$

$$\widetilde{f}_n(z_1) \in \ker(\widetilde{M}_n) \ \text{ and } \ \widetilde{f}_n(z_2) \in \ker(\widetilde{M}_n)$$

Conversely, if $\bigcap_{n\geq 1} \ker[M_n f_n] \subset \bigcap_{n\geq 1} \ker[\widetilde{M}_n \widetilde{f}_n]$, we have obviously
$\ker(\mathcal{B}^* S^*(.)) \subset \ker(\widetilde{\mathcal{B}}^* \widetilde{S}^*(.))$. □

In the case where

$$r_n = s_n \text{ and } \psi_{nj} = \varphi_{nj} \text{ for } 1 \leq j \leq r_n \text{ and } n \geq 1 \tag{4.22}$$

in particular if $\widetilde{A} = A$, then $f_n = \widetilde{f}_n$ for $n \geq 1$. As mentioned before, (4.22) may be verified even if $\widetilde{A} \neq A$. In such a situation, we have the following more practical result.

Corollary 24. *The system* $(\mathcal{S})$ *dominates* $(\widetilde{\mathcal{S}})$ *weakly on any interval* $[0, T]$, *if and only if* $\ker[M_n f_n] \subset \ker[\widetilde{M}_n f_n] \ ; \ \forall n$ 1, *or equivalently*

$$\ker[M_n] \subset \ker[\widetilde{M}_n] \ ; \ \forall n \geq 1$$

In the general case, we give hereafter characterizations of exact and weak domination for hyperbolic systems with respect to an output operator C.

Proposition 25. *The system $(\mathcal{S})$ dominates $(\widetilde{\mathcal{S}})$ exactly on $[0,T]$, with respect to C, if and only if, there exists $\alpha > 0$ such that for any $\theta \equiv (\theta_1,\theta_2)^{tr} \in Y$, we have*

$$\left\| \begin{pmatrix} \sum_{n\geq 1}[(\sum_{j=1}^{s_n} \sqrt{-\gamma_n}\langle C_1^*\theta_1,\psi_{nj}\rangle_\Omega \langle \widetilde{g}_i,\psi_{nj}\rangle_{1\leq i\leq q})\sin(\sqrt{-\gamma_n}t) \\ +(\sum_{j=1}^{r_n}\langle C_2^*\theta_2,\psi_{nj}\rangle_\Omega\langle\widetilde{g}_i,\psi_{nj}\rangle)\cos(\sqrt{-\gamma_n}t)] \end{pmatrix} \right\|_{L^2(0,T;\mathbb{R}^q)}$$

$$\leq \alpha \left\| \begin{pmatrix} \sum_{n\geq 1}[(\sum_{j=1}^{r_n} \sqrt{-\lambda_n}\langle C_1^*\theta_1,\varphi_{nj}\rangle_\Omega \langle g_i,\varphi_{nj}\rangle_{1\leq i\leq p})\sin(\sqrt{-\lambda_n}t) \\ +(\sum_{j=1}^{r_n}\langle C_2^*\theta_2,\varphi_{nj}\rangle_\Omega\langle g_i,\varphi_{nj}\rangle)\cos(\sqrt{-\lambda_n}t)] \end{pmatrix} \right\|_{L^2(0,T;\mathbb{R}^p)}$$

Proof. Derives from proposition 20. □

Proposition 26. *The system $(\mathcal{S})$ dominates $(\widetilde{\mathcal{S}})$ weakly with respect to C, on any interval $[0,T]$, if and only if,*

$$\bigcap_{n\geq 1}\ker[M_nP_{n,k}] \subset \bigcap_{n\geq 1}\ker[\widetilde{M}_n\widetilde{P}_{n,k}];\quad k=1,2$$

$$\textit{where } P_{n,k}(\theta) = \left(\ \langle C_k^*\theta_k,\varphi_{nj}\rangle\ \right)_{j=1,\cdots,r_n} \textit{ and } \widetilde{P}_{n,k}(\theta) = \left(\ \langle C_k^*\theta_k,\psi_{nj}\rangle\ \right)_{j=1,\cdots,s_n}$$

Proof. We assume that $\ker(B^*S^*(.)C^*) \subset \ker(\widetilde{B}^*\widetilde{S}^*(.)C^*)$, therefore, $\theta \equiv (\theta_1,\theta_2)^{tr} \in \ker(B^*S^*(.)C^*)$ if and only if,

$$\begin{aligned}&\sum_{n\geq 1}[(\sum_{j=1}^{r_n}\sqrt{-\lambda_n}\langle C_1^*\theta_1,\varphi_{nj}\rangle_\Omega\langle g_i,\varphi_{nj}\rangle)\sin(\sqrt{-\lambda_n}t)\\ &+(\sum_{j=1}^{r_n}\langle C_2^*\theta_2,\varphi_{nj}\rangle_\Omega\langle g_i,\varphi_{nj}\rangle)\cos(\sqrt{-\lambda_n}t)] = 0, \quad \text{for } i\in\{1,\cdots,p\};\ \forall\, t>0\end{aligned}$$

Using lemma 22, we have for $n \geq 1$ and $i \in \{1,\cdots,p\}$

$$\sum_{j=1}^{r_n}\langle C_1^*\theta_1,\varphi_{nj}\rangle_\Omega\langle g_i,\varphi_{nj}\rangle = 0 \quad \text{and} \quad \sum_{j=1}^{r_n}\langle C_2^*\theta_2,\varphi_{nj}\rangle_\Omega\langle g_i,\varphi_{nj}\rangle = 0$$

i.e. $\forall n \geq 1$, we have $P_{n,k}(\theta) = (\langle C_k^*\theta_k,\varphi_{nj}\rangle)_{j=1,\cdots,r_n} \in \ker(M_n),\ \ k=1,2.$
Using the same method for $\theta \equiv (\theta_1,\theta_2)^{tr} \in \ker(\widetilde{B}^*\widetilde{S}^*(.)C^*)$, we deduce that
$[\forall n \geq 1,\ k=1,2;\ \text{ we have } \ (\langle C_k^*\theta_k,\varphi_{nj}\rangle)_{j=1,\cdots,r_n} \in \ker(M_n)]$ implies that
$[\forall n \geq 1,\ k=1,2;\ \text{ we have } \ (\langle C_k^*\theta_k,\psi_{nj}\rangle)_{j=1,\cdots,s_n} \in \ker(\widetilde{M}_n)]$.
We then have the result. □

We consider now the case of multi-sensors.

4.4.2. Case of sensors

Now, if the output of the system is given by q_1 and q_2 sensors $(D_i,h_i)_{1\leq i\leq q_1}$ and $(D_i',k_i)_{1\leq i\leq q_2}$, we have
$C_1^*\theta_1 = \sum_{l=1}^{q_1}\theta_1^l h_l$ *and* $C_2^*\theta_2 = \sum_{l=1}^{q_2}\theta_2^l k_l$ for $\theta_1 = (\theta_1^l)_{l=1,\cdots,q_1} \in \mathbb{R}^{q_1}$ and $\theta_2 = (\theta_2^l)_{l=1,\cdots,q_2} \in \mathbb{R}^{q_2}$.
In this case, the exact and weak domination are equivalent. The following result gives a necessary and sufficient condition for the corresponding domination.

Proposition 27. *The system $(\mathcal{S})$ dominates the system $(\widetilde{\mathcal{S}})$ on any interval $[0,T]$, with respect to the sensors $(D_i,h_i)_{1\leq i\leq q_1}$ and $(D_i',k_i)_{1\leq i\leq q_2}$, if and only if*

$$\bigcap_{n\geq 1}\ker(M_nG_{n,k}^{tr}) \subset \bigcap_{n\geq 1}\ker(\widetilde{M}_n\widetilde{G}_{n,k}^{tr});\quad k=1,2 \tag{4.23}$$

where, for $k = 1, 2$; $G_{n,k}$ and $\widetilde{G}_{n,k}$ are the observability matrices defined by

$$G_{n,1} = \big(\ (\langle h_l, \varphi_{nj} \rangle) \ \big)_{\{l=1,\cdots,q_1\};\{j=1,\cdots,r_n\}}$$

$$G_{n,2} = \big(\ (\langle k_l, \varphi_{nj} \rangle) \ \big)_{\{l=1,\cdots,q_2\};\{j=1,\cdots,r_n\}}$$

$$\widetilde{G}_{n,1} = \big(\ (\langle h_l, \psi_{nj} \rangle) \ \big)_{\{l=1,\cdots,q_1\};\{j=1,\cdots,s_n\}}$$

$$\widetilde{G}_{n,2} = \big(\ (\langle k_l, \psi_{nj} \rangle) \ \big)_{\{l=1,\cdots,q_2\};\{j=1,\cdots,s_n\}}$$

Proof. Let us first note that

$$\mathcal{B}^* S^*(t) C^* \theta = \begin{pmatrix} \sum\limits_{n\geq 1} [(\sum\limits_{j=1}^{r_n} \sum\limits_{l=1}^{q_1} \theta_1^l \sqrt{-\lambda_n} \langle h_l, \varphi_{nj} \rangle_\Omega \langle g_i, \varphi_{nj} \rangle_{1\leq i\leq p}) \sin(\sqrt{-\lambda_n} t) \\ +(\sum\limits_{j=1}^{r_n} \sum\limits_{l=1}^{q_2} \theta_2^l \langle k_l, \varphi_{nj} \rangle_\Omega \langle g_i, \varphi_{nj} \rangle_{1\leq i\leq p}) \cos(\sqrt{-\lambda_n} t)] \end{pmatrix}_{1\leq i\leq p}$$

$$\text{and } \widetilde{\mathcal{B}}^* \widetilde{S}^*(t) C^* \theta = \begin{pmatrix} \sum\limits_{n\geq 1} [(\sum\limits_{j=1}^{s_n} \sum\limits_{l=1}^{q_1} \theta_1^l \sqrt{-\gamma_n} \langle h_l, \psi_{nj} \rangle_\Omega \langle \widetilde{g}_i, \psi_{nj} \rangle_{1\leq i\leq p}) \sin(\sqrt{-\gamma_n} t) \\ +(\sum\limits_{j=1}^{s_n} \sum\limits_{l=1}^{q_2} \theta_2^l \langle k_l, \psi_{nj} \rangle_\Omega \langle \widetilde{g}_i, \psi_{nj} \rangle_{1\leq i\leq q}) \cos(\sqrt{-\gamma_n} t)] \end{pmatrix}_{1\leq i\leq p}$$

Then the system $(\mathcal{S})$ dominates the system $(\widetilde{\mathcal{S}})$ on any interval $[0, T]$, with respect to C, if and only if $\ker(\mathcal{B}^* S^*(.) C^*) \subset \ker(\widetilde{\mathcal{B}}^* \widetilde{S}^*(.) C^*)$, then $\theta \equiv (\theta_1, \theta_2)^{tr} \in \ker(\mathcal{B}^* S^*(.) C^*)$ if and only if,

$$\sum_{n\geq 1} (\sum_{j=1}^{r_n} \sum_{l=1}^{q_1} \theta_1^l \sqrt{-\lambda_n} \langle h_l, \varphi_{nj} \rangle_\Omega \langle g_i, \varphi_{nj} \rangle_{1\leq i\leq p}) \sin(\sqrt{-\lambda_n} t) = 0; \ \forall t > 0$$

$$\text{and } \sum_{n\geq 1} (\sum_{j=1}^{r_n} \sum_{l=1}^{q_2} \theta_2^l \langle k_l, \varphi_{nj} \rangle_\Omega \langle g_i, \varphi_{nj} \rangle_{1\leq i\leq p}) \cos(\sqrt{-\lambda_n} t) = 0; \ \forall t > 0$$

we deduce that, $\forall n \geq 1$

$$\sum_{j=1}^{r_n} \sum_{l=1}^{q_1} \theta_1^l \langle h_l, \varphi_{nj} \rangle_\Omega \langle g_i, \varphi_{nj} \rangle_{1\leq i\leq p} = 0 \text{ and } \sum_{j=1}^{r_n} \sum_{l=1}^{q_2} \theta_2^l \langle k_l, \varphi_{nj} \rangle_\Omega \langle g_i, \varphi_{nj} \rangle_{1\leq i\leq p} = 0$$

Therefore, $\theta \in \ker(\mathcal{B}^* S^*(.) C^*)$, implies that

$$\theta_1 \in \bigcap_{n\geq 1} \ker(M_n G_{n,1}^{tr}) \ \text{ and } \ \theta_2 \in \bigcap_{n\geq 1} \ker(M_n G_{n,2}^{tr})$$

and hence (using analogous developments for $\theta \in \ker(\widetilde{\mathcal{B}}^* \widetilde{\mathcal{S}}^*(.) C^*)$)

$$\bigcap_{n\geq 1} \ker(M_n G_{n,1}^{tr}) \subset \bigcap_{n\geq 1} \ker(\widetilde{M}_n \widetilde{G}_{n,1}^{tr}) \text{ and } \bigcap_{n\geq 1} \ker(M_n G_{n,2}^{tr}) \subset \bigcap_{n\geq 1} \ker(\widetilde{M}_n \widetilde{G}_{n,2}^{tr})$$

Consequently, we have the result. □

4.5. Application to the one dimension wave equation

To illustrate the previous results, we consider without loss of generality, a one dimension hyperbolic system described by the following wave equation,

$$(\mathcal{S}_g) \begin{cases} \dfrac{\partial^2 x}{\partial t^2}(\xi, t) &= \dfrac{\partial^2 x}{\partial \xi^2}(\xi, t) + g(\xi) u(t) &]0,1[\times]0,T[\\ x(\xi, 0) &= \dfrac{\partial x}{\partial t}(\xi, 0) = 0 &]0,1[\\ x(0,t) &= x(1,t) = 0 &]0,T[\end{cases}$$

with $\Omega =]0,1[$. The operator A is defined by $A\ f = \dfrac{\partial^2 f}{\partial \xi^2}$, with

$$\begin{aligned} D(A) &= \{f \in L^2(0,1)/\ f,\ \frac{\partial f}{\partial \xi} \text{ absolutely continuous}, \\ &\qquad \frac{\partial^2 f}{\partial \xi^2} \in L^2(0,1) \text{ and } f(0) = f(1) = 0\} \end{aligned}$$

A is self-adjoint and $(-A)$ is positive. Moreover, the eigenvalues of A are $\lambda_n = -n^2\pi,\ n \geq 1$. The corresponding eigenfunctions are defined by $\varphi_n(\xi) = \sqrt{2}\sin(n\pi\xi)$. The operator $\mathcal{A} = \begin{pmatrix} 0 & I \\ A & 0 \end{pmatrix}$ generates a s.c.s.g. defined by

$$S(t)\begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} \displaystyle\sum_{n=1}^{+\infty}[\langle z_1, \varphi_n\rangle \cos(n\pi t) + \frac{1}{n\pi}\langle z_2, \varphi_n\rangle \sin(n\pi t)]\varphi_n \\ \displaystyle\sum_{n=1}^{+\infty}[-n\pi\langle z_1, \varphi_n\rangle \sin(n\pi t) + \langle z_2, \varphi_n\rangle \cos(n\pi t)]\varphi_n \end{pmatrix}$$

In this case, it is sufficient to consider an interval $[0,T]$ with $T \geq 2$. $(\mathcal{S}_g)$ is excited by a zone actuator (Ω, g), then we have $U = \mathbb{R}$ and $Bu(t) = g(.)u(t)$.
The considered system is augmented with following output equation

$$y(t) = Cz(t) \text{ where } z(t) = \begin{pmatrix} x(.,t) \\ \frac{\partial x(.,t)}{\partial t} \end{pmatrix}$$

The system $(\mathcal{S}_{\widetilde{g}})$ denotes the system $(\mathcal{S}_g)$ by replacing g by $\widetilde{g}$. The characterizations of exact and weak domination remain true on any interval $[0,T]$ with $T \geq 2$. They may be deduced easily from the previous results with $\widetilde{A} = A$, $r_n = s_n = 1$ and $\varphi_n = \psi_n$.

In the particular case where $C = \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix}$, we have the following result.

Proposition 28.
The system $(\mathcal{S}_g)$ dominates the system $(\mathcal{S}_{\widetilde{g}})$ weakly on $[0,T]$, if and only if, for any $n \geq 1$, we have

$$\langle g, \varphi_n\rangle = 0 \Longrightarrow \langle \widetilde{g}, \varphi_n\rangle = 0$$

Now, if $(\mathcal{S}_g)$ and $(\mathcal{S}_{\widetilde{g}})$ are augmented with the output equation

$$y(t) = \begin{pmatrix} \langle h, x(.,t)\rangle \\ \langle k, \frac{\partial x}{\partial t}(.,t)\rangle \end{pmatrix}$$

using proposition 26, we deduce the following characterization.

Proposition 29.
The actuator (Ω, g) dominates the actuator $(\Omega, \widetilde{g})$ with respect to the considered sensors, if and only if

$$\forall n \geq 1 \left\{ \begin{array}{l} \langle g, \varphi_n\rangle\langle h, \varphi_n\rangle = 0 \\ \langle g, \varphi_n\rangle\langle k, \varphi_n\rangle = 0 \end{array} \right. \Longrightarrow \forall n \geq 1 \left\{ \begin{array}{l} \langle \widetilde{g}, \varphi_n\rangle\langle h, \varphi_n\rangle = 0 \\ \langle \widetilde{g}, \varphi_n\rangle\langle k, \varphi_n\rangle = 0 \end{array} \right. \tag{4.24}$$

These results can be also applied to a two-dimension space domain (a rectangle for example) and extended to more general situations and systems.

5. Conclusion

In this paper, we show how distributed parameter systems can be classified using the concept of domination. This work is an extension of this concept to general classes of distributed systems, which dynamics may be described by different operators. It has been explored for general classes of controlled systems and input operators. Characterization results and main properties are presented. By duality similar results are established for observed systems and output operators.

Two main classes of physical systems have been considered, parabolic (diffusion) and hyperbolic systems (wave equation). The case where the system is excited by multi-actuators is also developed. Applications an illustrative examples are presented. The obtained results give a mean to compare and classify such systems, as well as input and output parameters and hence permit to make better choices.
The domination can be extended to more general systems (delayed systems, non linear and stochastic systems). The case where the objective is given by a more complex cost function could also be explored making the domination concept useful for any situation. These prospects are considered and will be published in future works.

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Revisiting the Pasteur Quadrant, Post-normal Science and Strategies for Research on Natural Hazards and Disasters

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Abstract:

The paper reviews two paradigmatic research strategy models: a) The Linear Research Model; a lineal one-dimensional plane model; comprising basic research at one end and applied research at the other and using the maxima that "*applied research invariably drives out from pure research". b)* The Quadrant Model of Scientific Research; a two dimensional Cartesian plane model **[1]** characterizing four research quadrants: i) The Bohr´s Quadrant, where the quest for fundamental understanding is high, but no so regarding apply research. ii) The Pasteur´s Quadrant, where both the quest for understanding and consideration for use of the information are high ("U*se-Inspired Basic Research or Mission Oriented Research Quadrant*"). iii) The Edison´s Quadrant, where the quest for fundamental research is rather low and there is high consideration for use of the information. iv) The Empty Quadrant, where there is no considerations either for developing fundamental knowledge or for use. It is argued that research planning dealing with natural hazards and mitigation of disasters (particularly in the developing world) has to consider the use of Stokes´ quadrants and particularly focus in the Pasteur´s Quadrant. Looking into comprehensive research and the conformation of high level inter-disciplinary teams, long term financing schemes and pointing towards the implementation of Post-normal science strategies, where the dissemination of knowledge and education to society and the use of local knowledge, are critical aspects. Finally, the case of the 2010 devastating earthquake and tsunami in Chile is used as an example where the existence of just outstanding Normal-science knowledge (i.e Bohr´s Quadrant) was not enough to confront it.

Developing countries need to invest in team-organized and modern planned research regarding their own natural hazard and disaster priorities.

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Keywords

Pasteur´s quadrat, hazard, impact, research, inter-discipline, post-normal science, Chile.

Introduction

What is called "Normal Science" has been defined **[2]** as: *"research firmly based upon one o more past scientific achievements; achievements that some particular scientific community acknowledges for a time as supplying the foundations for its further practice"*. If these achievements are sufficiently unprecedented and attract an enduring group of scientist away from other unprecedented achievements, and if they are sufficiently open-ended to leave problems to be redefined, they are called by Kuhn "science paradigms". There is a beautifully description of the way in which old science paradigms are replaced by new ones, previous to the presence of a "sense of malfunction"; a prerequisite for scientific revolution; described as punctuated interludes, where scientific paradigms are been sequentially replaced **[2]**. Some of the main and initial scientific revolutions occurred around the nineteenth century that reinforced in the first half of twenty century, greatly helped to shape what we use to call the modern-world or the world where material progress has occurred at a comparative faster rate. Philosopher of the sciences and political scientists have scrutinized not only in the structure of such scientific revolutions, but also with regards to the associated scientific policies schemes implemented by leading developed countries, particularly after Word War II and at the end of the so called cold war. Donald E. Stokes **[1]** was one of such political scientists deeply thoughtful about policy processes and that deeply understood science. Stokes´ book "Pasteur´s Quadrant: Basic Science and Technological Innovations" is a land mark on the matter. On the other hand, Post–normal science (at the beginning much linked to ecological economics) is a truly new conception of the management of complex science-societal related issues **[3, 4, 5, 6]** , advocating the inclusion and integration of knowledge derived from science, but also addressing: a) uncertainty, b) value loading, c) plurality and importantly c) communication of science to society. In other words Post-normal science addresses the need to focus on complex societal-problem-solving via the coupling of science together with the relevance of human commitments and values and face epistemology as well as governance issues. This approach goes over and above concepts used in the literature such as integrative, interdisciplinary, transdisciplinary or landscape research **[7]** .

The suddenness and unpredictability of natural hazards (= naturally occurring physical phenomena caused by either rapid or slow onset of events having atmospheric, geologic and hydrologic origins on solar global, regional, national or local scales; definition according to UNESCO) such as earthquakes, tsunamis, flooding, landslides, droughts; or for the matter human-driven step by step building processes leading to earth systems towards tipping points **[8]** , such as climate change, global warming, water shortages and associated risks that seriously and dramatically affect nations. These are indeed critical research areas that urgently need to be approached via post-normal science, complemented or assisted with the so called Pasteur´s Quadrant, Research Strategy. To accomplish that, a country needs to develop well thought research plans regarding own research on natural catastrophes priorities and to allocate long-term resources. In this regards, science and technology academies, like the Kingdom of Morocco, Hassan II Academy of Science and Technology, may play a crucial role in it.

Revisiting Pasteur´s Quadrant

Around the world, governments and research institutions have approached the planning and financing of Normal science research from different angles and using various models. Perhaps two of the better known models are:

a) **The Linear Research Model** (a lineal one-dimensional plane model), comprising basic research at one end and applied research at the other **[1, 9]**. The structure of the model is that linearly basic research, leads to applied research, resulting in development and eventually into production, operations and technological innovations **[1]** . In this model, each of the successive stages depends upon the preceding one/ones and were described in "Science-The Endless Frontier: A Report to the President on a Program for Postwar Scientific Research": *"applied research invariably drives out pure research",* what is known as Bush´s perverse law governing research **[9]** . Here, the main paradigm is that there is a tension between curiosity-driven basic research that is performed without thought of practical use or end and applied research, which tends to be planned and developed directed toward some individual, groups or users. In fact, *"basic research is the pacemaker of technological progress"* **[9]** . In USA Science was: Science --The Endless Frontier Program, and five years later lead to the creation of the National Science Foundation, a powerful research funding agency that it was thought and established independently, as much as possible, from political control **[1]** .

b) **The Quadrant Model of Scientific Research** (a two dimensional Cartesian plane model) [1] (Fig 1). In this model the vertical axis represents the degree to which a given body of research seeks to extend the frontier of fundamental understanding, while the horizontal axis represents the degree to which the research is guided by considerations of use. Stokes **[1]** characterized four main research type of quadrants: a) the Empty Quadrant at bottom left-hand quadrant, where there is no considerations either for developing fundamental knowledge or consideration for use; b) The Bohr´s Quadrant ate the upper left-hand, where the quest for fundamental understanding is high, but no so regarding considerations for use of the information; c) The Edison´s Quadrant, at the bottom right-hand, where the quest for fundamental research is rather low, by high the consideration for use of the information; d) The Pasteur´s Quadrant, at the upper right-hand, where both the quest for understanding and consideration for use of the information are high. This quadrant can be labeled as the "Use-Inspired Basic Research Quadrant" and also as the: Applied, Mission Oriented or Service Oriented Research Quadrant **[1]** .

In Stokes´ book **[1]** is found a full description about how dissents (researchers, agencies, countries) of the Bush´s one-dimensional Linear Research Model **[9]** developed different research models away from the linear one–dimensional model, for the planning, implementation and the creation of new research agencies or national research structures for the allocation of basic, applied or mixed research funds. All of them recognizing a more complex relationship between the quest for fundamental understanding and considerations for the use of the information (applied research), aiming to bridge research and industrial development according to country priorities. Nevertheless, no abandoning the idea that curiosity-driven basic research is a key part of the puzzle.

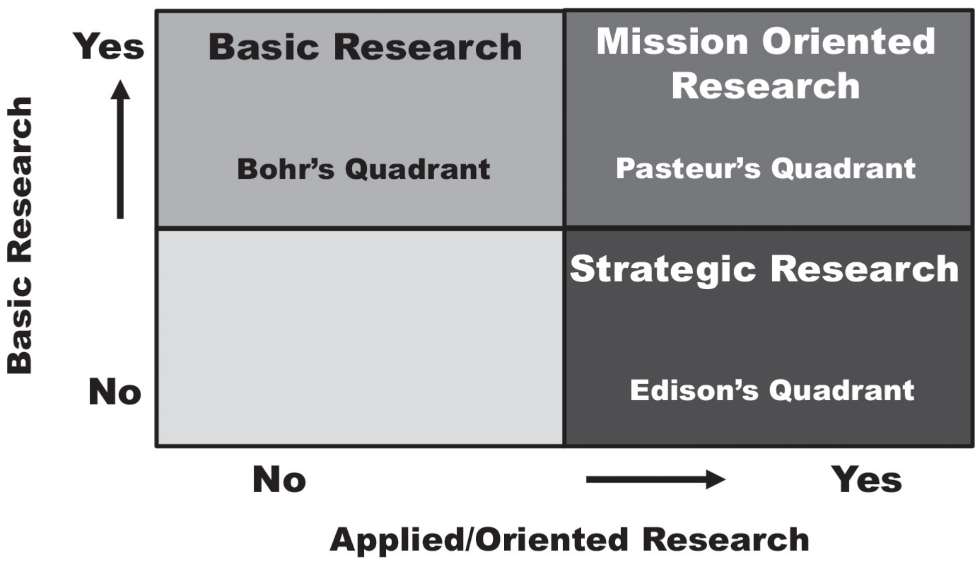


Figure 1. Stokes`Quadrant Model of Scientific Research. (Taken and modified from Stokes **[1]**).

Hereby, I pose that in many circumstances there is a need for countries to prioritize and allocate adequate long-term funds for research on natural hazards, associated risks and disasters. The strategies have to integrate interdisiciplinary teams aligned with the Mission Oriented Pasteur Quadrant. By this I mean making sure that the fundamental research to deal with natural hazards and disasters is duly developed inside the country (-obviously taken advantage of global produced knowledge-), and is the same time that advanced knowledge is indeed of high scientific quality (-judged for instance for the level, quality and impacts of publications-). But, at the same time, importantly considering the transference of the information to society and enhancing governance schemes. Here, is exactly the space to integrate a "Pasteur Mission Oriented Research Strategy" on natural hazard and disaster and associated risks in connection with the relevance of such knowledge to human communities, values and governance issues. This is to say, moving into the direction of the new paradigm labeled as Post-normal science.

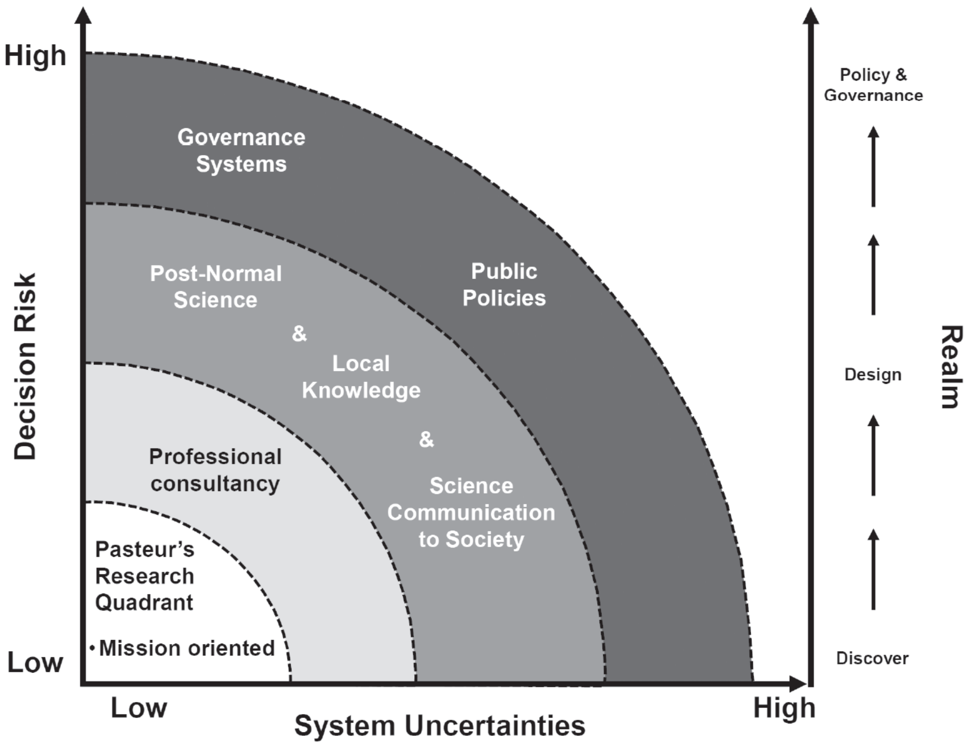
Visiting Post-modernism and Post-normal science

The so called pre-modern era was one where Normal science (see above) was practically unknown and religion and traditional knowledge were the main sources of truth and explanation of reality. In the modern era, step by step, science became the predominate source for truth to explain nature and reality. The Sun, Earth, the age of planet Earth and *hominin* fossil may serve as an example. Copernicus, a Catholic cleric, in 1543 argued that the Sun, not the Earth, was the center of the Universe, this against the well accepted opposite general believe. In 1633, Galileo was placed under house arrest by the Inquisition for endorsing this theory. At that time it was overwhelmingly accepted that the planet Earth was created around 6000 years ago. Actually, English speaking Christians accepted that God had created Earth on October 23, 4000 B.C. **[10]** . In 1778 there were proofs than indeed planet Earth was significantly older than 6000 years and that in fact its age was incalculable: it could be hundreds, millions of billions of years **[10]** . Furthermore, Darwin **[11]** , heavily influenced by Hutton´s theories**[10]** , sustained that *Homo sapiens* was descendent from an ancestor shared with the common ape. In the past 180 years we have unearthed 27 separate *hominin* fossils (-ancient and modern humans) that split of from a common chimp-like ancestor, around 7 millions years ago. Surely, 26 of them are not longer with us, but we know amazing details about their anatomies, physiologies, behaviors and about their scattering around the world. We are the last survivors of the 27 variety of *hominin* so far discovered on Earth **[12]** .

The list of scientific advances and discoveries in this and many other areas is endless. Nevertheless, narratives as those of the Scottish Enlightenment times are definitely gone. At present *"knowledge is constructed, not discovered, its is contextual, not foundational"* **[13]** . Today, we find ourselves in a post-modern world, or in the Anthropocene era **[14]** , that lacks powerful narratives and where science is not, necessarily, any longer the past gold fountain for the explanation of realities. Further, where the cultural constructions perhaps still depends on some science, but where influences are much smaller of physical sciences, much larger in the social sciences and lesser in pure than in applied science **[6]** . Hence, Post-normal science is a new conception of the management of complex science-social related issues and it focus more on aspects of problem-solving heavily impregnated by uncertainty and risks **[15, 16]** , values loading and plurality of societal legitimate perspectives **[3, 4, 5]** . Post- modernity is the time of science-societal complexities and uncertainty of human commitments and values and one marked by deep human-driven changes on earth and sustainability. Concurrently, is the time for Post-normal science, that necessarily advocates the inclusion and integration of knowledge derived from science, but now addressing, uncertainty, value loading, plurality and most important than all, the communication of science to society. The science ivory tower paradigm anchored in universities is over. It has been replaced by a complex and entangling combination of science-societal fundamental and mission oriented applied science, going through professional consultancy (if necessarily) and flowing through the turbulent waters of Post-normal science with value loading, science communication to society, and ideally reaching policy spheres and governance schemes (Fig. 2). Not a minor task.

Figure 2. Problem solving diagram .When uncertainties and risk are low, we are in the Normal-science

"discover" realm. In the middle, most often scientific expertise will be not enough and there would be a need for professional consultancies and elements of Post-Normal science; we are in the "design" realm. When risk and uncertainties are high we are in the "policy and governance" realm. Here, defensive strategies could appear, challenging even advanced knowledge. (Taken and modified from **[4]**).



Natural hazard, disaster, mission oriented research and the role of Post-normal science

According to UNESCO, natural disasters are the combination of hazards conditions of vulnerability and insufficient capacity or measures to reduce the potential negative consequences of risk. Nevertheless, natural disasters are not necessarily entirely "natural", since people can be active local or global agents of disasters (environmental human-driven hazards and disasters). Hence, flooding may be exacerbate by deforestation or the improper management of water sheds. Climate change, global warming and modifications of the carbon cycle are undoubtedly global hazards been driven by humans **[17, 18, 19]** . Not all, by many natural disasters can be greatly mitigated. As proposed by UNESCO International Strategy for Disaster Reduction, mitigations include the fields of risk awareness and assessment, building of local and global knowledge, public information, commitments and the development of appropriate institutional frameworks; as well as early warning systems including forecasting, dissemination of warnings, preparedness measures, reaction capacities and above all education. At a national level the approach to theses complex interdisciplinary fields require the adoption of forethought planning strategies at different institutional levels. There are several fields of knowledge that do require basic information (Normal-science approaches) related to different geo-bio-physical as well as socio-economic disciplines. At the nation-level research planning to deal with natural hazards and to mitigate natural disasters is one of the most fruitful field for the developing of Mission Oriented Team of Researchers (see above) and for the implementation of Post-normal science strategies, where the dissemination of knowledge and education to society, as well as the use of local knowledge, are critical aspects. It will not just be a matter of developing more up-to-day information (that is much needed), but moreover developing/inventig new tools and mechanisms to address how to tackle complex science-social potential hazards and disaster issues impregnated by uncertainty and risks.

In this regards, the February,27th, 2010 Chilean maga-earthquake and tsunami disaster is a case worth to address. The earthquake occurred at around 4 AM and had a magnitude 8,8 MW (the fourth largest even recorded with instruments in the world **[20]**. Normal-science had predicted in 2009, that concerning the seabed fault in Chile along 35-37° S: " *…, in the worse case scenario*, *the area already has a potential for an earthquake of magnitude as large as 8-8,5, [and] should it happen in the near future"* **[21]**. Obviously, the exact date of the earthquake was not predicted. But the amount of accumulated energy in the geo-fault area, since the previous large quake in the same area in 1835 (-witnessed by Charles Darwin during his stay in Chile **[22]**, make the scientific prediction a very valid one. Indeed, this was exactly the area where the earthquake and tsunami occurred in February 2010. It can be stated that a group of scientists **[21, 23, 24]** made a proper job from a geo-physical Normal-science perspective, but unfortunately, they were no part of a integrate and comprehensive team dealing with complex earthquake and tsunami hazards in Chile, that would have the possibility to use that information in a more comprehensive and science-societal perspective. As basic scientists they just made a good job; they produced outstanding science and published in good international scientific journals. The lesson (in short) is that Normal-science is just one part (a very important one indeed!) of a Mission Oriented Research Team or a Post-normal Research Team, dealing with hazards, disasters and risks, such as earthquake or tsunamis. Nevertheless, it may be consider that regarding the occurrence of natural hazards and disasters… the population preparedness is at stake. In Chile, at the time, that mission oriented research strategy, and proper allocation of resources, did not exist. It is no possible (or proper) to speculate if the existence of such teams in Chile would had helped or nor to a better preparedness of the population, mitigating in that way the impacts and fatalities (but see **[25]**); nevertheless, the international experience signs in that direction. It seems to me that in this respect the Chilean government has learnt some lessons, since in the past two years a special Team-research Program has been developed and funded to deal with natural hazards and disasters in the country. This is a step forward in the right direction.

Conclusions

Research on natural hazards and disasters represent critical challengers for governments and particularly for the developing world. On the matter, long-term strategic research polices need to be implemented and priories to be set. Concurrently, long-term and adequate funding are absolutely essential. For instance, natural hazards (real natural ones or human-driven ones) may seriously affect economy of countries and societies as a whole for climate change **[18]**. The so called Normal-science is absolutely necessary to advance in generating curiosity-driven knowledge in these arenas, but not sufficient. Natural hazards encircle the management of complex science-societal related issues, and national research policies, strategies and tools such a those proposed by Stokes **[1]**, regarding the application of a bi-dimensional Quadrant Model of Scientific Research, rather than the usual one-dimensional Linear Research Model is worth to consider. In the tackling of national research on natural hazard, risks and disasters a mission team-research oriented strategy is suggested. Furthermore, post- modernity, and the era of the Anthropocene in which we live, calls for science-societal Post-normal science integrated approaches. Hazards and disasters are extremely complex research arenas. So Post-normal science approaches advocate that the knowledge derived from transdisciplinary scientific approaches, including, uncertainty, value loading and plurality needs to be communicated to society. Eventually the processes should lead into the direction of public policies and governance spheres. The case of the 2010 devastating earthquake and tsunami in Chile is a good example where the existence of just outstanding Normal-science knowledge, on a critical recurrent hazard for the country (earthquakes), was not enough to confront it. From this and other examples, lessons need to be derived: the costs involved in research leading to the mitigation of natural hazards and disasters are, generally speaking, small as compare to the costs of relief, recovery and lost of lives. Developing countries need to invest in well team-organized and modern planned research with regards to own natural hazard and disaster priorities.

There is most often the perception that tackling mission-oriented work will reduce the quality of research.

Nevertheless, the experience of Japan, USA and Australian indicates the contrary. For instance, in Australia the increase focus on mission-oriented research (mostly via the Commonwealth Scientific and Industrial Research Organization, CSIRO) has indeed increase Australia's ranking as one of the leading source of intellectual property **[26]**

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Natural and societal-induced environmental hazards: integrate interdisciplinary long–term research strategy for developing countries *

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Abstract

Natural stochastic and societal-induced hazard events (S-IHEs), such as meteorological, climate, hydrological, geophysical and biological, are part of the so called *"science of natural and societal-induced hazards and disaster risk"*. In the past 50 years world impacts due to natural and NS-IHEs have increased about one order of magnitude, showing severe increases in economical damages. Moreover, over 90% of the population affected by them refers to events such as flooding, windstorms and droughts, with a mean of about 200,000 people directly affected per year. In my view the "hard-science" behind natural and S-IHEs involve basic disciplinary research as well as in the so called mission integrative and multidisciplinary research. In the are of natural hazards and S-IHEs there is an urgent need for disciplines to truly "talk-each-other" in an integrative way. Chile is a developing country facing numerous and dreadful natural and S-IHEs and therefore, scientific research (preparation, response, recovery, mitigation) and linkages with policy making and government, need to be part of integrate interdisciplinary long–term scientific research strategies. The paper describes, with details, a research multidisciplinary initiative (FONDAP Programs) highlighting long-term results regarding first world class publications, that may serve as an example for building natural and S-IHEs investigative and the design of public policy strategies in other developing countries.

Keywords:

Environmental hazards, disasters, integrative multidisciplinary research, mission oriented research model, developing countries, Chile.

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1. Introduction and definitions

Natural stochastic and societal-induced hazard events (S-IHEs), some times sudden and violent and extreme in magnitude such as: a) meteorological (wind-storms, typhoons, hurricanes extreme temperatures); b) climate (droughts, wildfires); c) hydrological (flooding, landslides); e) geophysical (earthquakes, volcano eruptions, tsunamis); f) biological (infectious diseases), are part of the so called *"science of natural and societal-induced hazards and disaster risk"* (other type are the so called technological impacts, such as those related to oil spills, explosions, transport , other). When these events cause severe damage to society are called disasters or catastrophes (in the last case referring to an extreme disaster event). According to the International Agreed Glossary of Basic Terms to Disaster Management [1] the definition for disaster is: *"Situation or event, which overwhelms local capacity, necessitating a request to national or international level for external assistance". A*lthough, natural disaster specialists (www.atlantisinireland.com/hazards.php) had suggested and alternative definition as following: *"A physical natural event (*and/or societal-induced event*) that kills people or overwhelms local capacity for damage control or recovery"* (parenthesis introduced by the author).

In this paper I have introduced the concept of societal-induced hazard event as a complement to fully natural hazard events. The goal is to call attention not only to the fact that there is an increase interaction between natural hazards and societal conditions (i.e. due to poverty, overpopulation, human modification of the environment, inadequate human settling areas, other), that has led to an every-day increase risky hazard-prone areas; but moreover, to the fact that the Fifth Assessment Report of the Intergovernmental Panel on Climate Change [2] reported that: *"The IPCC is now 95% certain that humans are the main cause of current global warming. In addition, the SYR finds that more human activities disrupt the climate, the greater the risks of severe, pervasive and irreversible impacts for people and ecosystems, and long lasting changes in all components of the climate system"*. This, indeed, is directly linked to several of the above listed recurrent "natural hazards", particularly those related with meteorological, hydrological, climate and biological factors. Furthermore, the IPCC [2] Report indicates: "a*nthropogenic green house gas emissions have increased since the pre-industrial era, driven largely by economic and population growth, and are now higher than ever. This has led to atmospheric concentrations of* CO_2*, methane and nitrous oxide that are unprecedented at least in the last 800,000 years. Their effects, together with those of other anthropogenic drivers, have been detected throughout the climate system and are extremely likely to have been the dominant cause of the observed warming since the mid-20 century"*.

So, today there is ***"a very likely probability"*** *(*= 90-95%; [2]) that many extreme weather and climate changes events are linked to societal influences. For instance: decreases in cold temperature extremes, increases in warm temperature extremes and abnormal heavy precipitations, have been observed in numerous regions of the world, and many of them are directly linked to flooding, waves of droughts, wildfires and landslides hazard events; ought to be considered as societal-induced or driven environmental hazard events.

Between 1960 and 2009 hazard impacts due to natural and NS-IHEs have increased from 450 in the decade 1960-1969 to 4308 in the decade 2000-2009; showing highly severe increases in economical damages (Fig.1). Moreover, for the period 1994-2013 over 90% of the population affected by these hazards refers to: a) flooding, b) windstorms, c) droughts; with a mean of about 200,000 people directly affected per year. For that period reports show over 1, 300, 000 people deaths; with over 50% referring to earthquakes and tsunamis [3, 4].

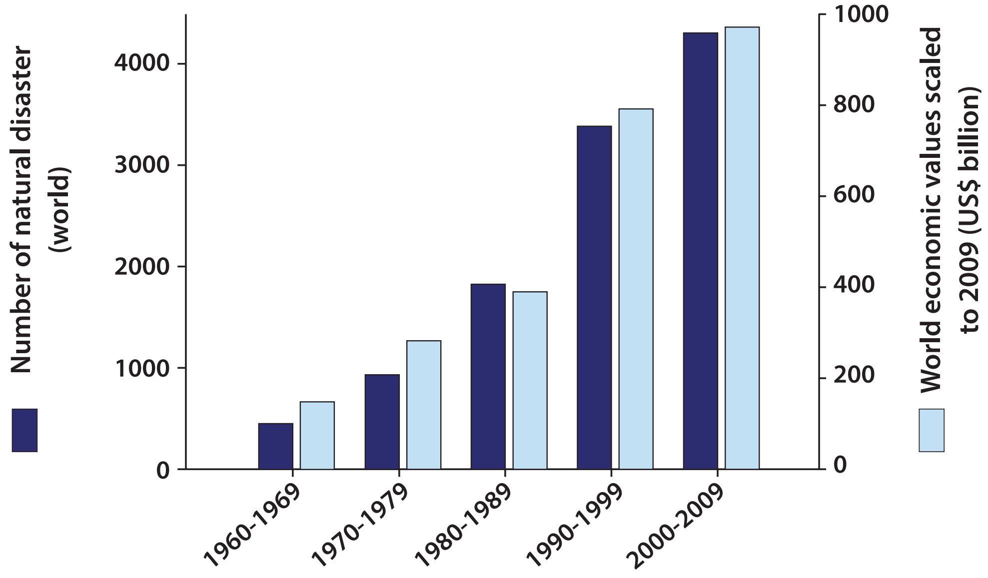

Figure 1. Total number of world natural disaster (meteorological, climate, geophysical and biological) grouped per decade and their economic values (losses) in US $, scaled in real prize to 2009 (information taken from [3]; scaled US $ to real prize from 1960 to 2009 by the author from www. measuringworth.com)

2. The scientific perspective of natural and S-IHE environmental hazard events

There are many possible approaches for the development of scientific research and linkages on environmental hazard studies and programs (academic, governmental, state, other institutions). They range, in one extreme, from extremely focal, specialized or disciplinary studies or, in the other extreme, to overarching multidisciplinary scientific approaches. In any case, most of international, and many regional or national strategies, research programs on environmental hazards tend to focus on disaster risks, disaster reductions, damage control and recovery [5]. Risks depend on the type and magnitude of the hazard event and on vulnerability (= the loss from natural or S-IHEs) and this makes environmental hazard research programs indeed a multidisciplinary discipline.

Among other, the discipline of risk analysis considers physical, biophysical, health, human and societal elements; incorporating humanities, education, social and natural sciences. This, includes the study, analysis and evaluation of the type and probability of hazard occurrence, the range of intensities, effects and modeling; as well as, human behavior, local ecological knowledge, societal impacts and importantly the political decision-making chain-process, from the hazard impact (ideally previous to the occurrence), all the way to control and recovery stages. Then, there is a need for integrated approaches, not only across disciplines but across spatial, temporal and the different levels of governance. Moreover, there is a critical need to be able to build strong data-bases on environmental hazards events [5].

In my view the “hard-science” behind natural and S-IHEs lays both in the Basic Research or Bohr´s Quadrant as well as in the Pasteur´s or Mission Oriented Quadrant [6, 7]. In fact, regarding environmental hazards events at the local scale (i.e. country or region) basic disciplinary science is much needed (i.e. geology, clime, oceanography, sociology, political science, economy, modeling, other). Nevertheless, interdisciplinary scientific approaches are critical and they can not be limited to the study of local situations but need to be integrated into wider national and international scientific approaches, and must be based on long–term research teams, strategies and funding [5].

Interdisciplinary research (overarching research on a particular problem or subject) is difficult to be achieved, since it usually considers both the quest for fundamental understanding as well as aspects aiming to bridge over applied and/or societal problems. The scientific programming on natural and S-IHEs is a case in point. But, also is the case for other societal problems or challengers, for instance interdisciplinary studies on: Biodiversity Losses and Impacts on Society; Sustainable Aquaculture and Feeding the Poor; The Challenge of Ageing or Drug Abuses and Society.

In developed countries, with long traditions and considerable research funding, interdisciplinary research, focusing on above examples and many other have flourished, particularly following the end of the Second World War [6]. Such appears not to be the case in many developing or emerging countries, where incentives for interdisciplinary research is absent or weak. Under those circumstances, and with limited research funding, the usual situation in these countries is one in which it may exist the development of several disciplinary sciences (depending on funding and trained scientific personnel), but where the disciplines "do not talk-each-other"; even if some of them are well developed. In this case research funding agencies (governments) need to develop new research strategies and above all to provide incentives.

3. A model to develop and incentive interdisciplinary studies: the Chilean strategy

In 1967 Chile, presently a country member of the Organization for Economic Co-operation and Development (OECD), started a plan to seriously develop science, technology, innovation and the training of human capital. That year it was established the Comisión Nacional de Ciencia y Tecnología (CONICYT) (National Commission for Scientific and Technological Research), under the Ministry of Education, as an advisory body to the President of the Republic. CONICYT mission is: *"advancing the training of human capital, and promoting, developing, and disseminating scientific and technological research….. aiming to contribute to Chile´s economy, social, and cultural development"*. CONICYT provides resources for highly competitive funding calls and creates opportunities for coordination, networking and designs strategies to implement scientific public awareness. Presently this agency administers a set of 12 different major programs, for example on: Equipment Funding, Science Divulgation and Fellowships (www.conicyt.cl).

Additionally, in 1982 was created the Fund for National Scientific and Technological Development (FONDECYT), that has financed several competitive research programs; out of which, one of the most important one is the so called "FONDECYT: Regular Scientific Program in all scientific disciplines: Natural, Cultural and Formal Sciences (www.fondecyt.cl), with financing windows of 2-4 years per project. Furthermore, this agency, manages a special fund that focus on long-term (up to 10 years per Center) interdisciplinary national scientific excellence and mission oriented scientific programs [6, 7], selecting Chilean priorities areas, call FONDAP "Fund for Research Centers on Priority Areas" (Fondo de Financiamiento de Investigaciones en Areas Prioritarias". FONDAP Programs started in 1997 and provide long-term funds for research centers of excellence for Chile´s development. The goal being to articulate teams of national outstanding researchers (selected on the basis of scientific productivity) in priority areas, aiming to consolidate them guided by interdisciplinary, national and international networking approaches and, centrally, aiming to the training of young scientists (human capital). A third Program is called FONDEF, directed to strategic-problem solving aspects of science, technology or industry, which is jointly financed with the industry (up to 4 years per project). The scientific policy behind these programs, especially behind the FONDAP initiative, is aiming for networking, first world class paper productivity and first world class personnel training [8, 9]. Selection of proposals and controls occur every 2-3 years under the responsibility of international panels of experts.

Among the main incentives for researchers engaged in FONDAP Centers are: a) Long-term temporal window of funding; initially 5 years and extended for another 5 years (based on international evaluations); b) Sustained funding of about 1- 1.5 million US dollars per year per Center; c) Economic incentives, on top of salaries, for researchers engaged in the Center, d) Interdisciplinary approaches, national and international networking, e) Funding for Ph. D and Post-doctoral young scientists, f) Funding for equipment, g) Incentives to access matching, national and international, research funds.

Since 1997 funds for 20 FONDAP Centers have been allocated in Chile, and in 2015 there are 11 FONDAP Centers in operation, with an annual budget of approximately 18 millions of US dollars. Examples of such Centers are: a) *"Multidisciplinary Center for Intercultural and Indigenous Studies"*; b) *Center for Climate Change and Resilience*; c) "*Center for Solar Energy Research*"; d) "*Center for Sustainable Urban Development"*; e) "*Interdisciplinary Center for Sustainable Aquaculture Research*", f) "*Center for Astrophysics*"; g) "***National Research Center for the Integrated Management of Natural Disasters***" (www.fondap.cl).

Furthermore, in 1999 it was initiated a second and similar set of Scientific Research Centers of Excellence, now under the Ministry of Economic Affairs, call the Millennium Scientific Institutes (MSI) and the Millennium Scientific Nucleus (MSN) [10]. Further, promoting outstanding research, the training and reinsertion of Chilean scientists. In the future, it is hope, that both initiatives, FONDAP and MILENIO, will be merged.

4. The FONDAP "Center National Research Center for the Integrated Management of Natural Disasters" (CIGIDEN)

CIGIDEN (2012-2017) is a recently financed FONDAP Center of excellence; in this case is based at the Pontificia Universidad Católica de Chile, School of Engineer, and has 3 other associated Chilean Universities. With a total of about 50 researchers and 6 main research lines; under 6 main principal investigators and 6 associated investigators. Some of research lines are: Surface waters; Disaster Risk Vulnerability-Physical and Sociological Systems; Management of Disaster and Risk Mitigation. The Center is linked to main national services (i.e. geology, seismology and climate) and Hazard-Disaster National Offices. One of the main objectives of CIGIDEN is to develop, integrate, and convey knowledge allowing the creation of a system that can respond effectively to extreme natural phenomena, achieved through the preparation, response, recovery, and mitigation stages. CIGIDEN is an integrating, interdisciplinary research initiative, which contributes to address the need of mitigating the impact of natural disasters on Chilean society, physical infrastructure, and economic development. CIGIDEN is the basis to generate new knowledge and technology that will enhance the understanding and mitigation of the global implications of natural and societal-induced disasters in the country, along with the establishment of territories that are less exposed and of communities that are more resilient (www. cigiden.cl).

5. Developing countries: science, technology investment, research strategy

Chile is an emerging country (*ca*. 20,000 US dollar per year, per capita), and member of the OEDC. In the area of Science and Technology (S&T) Chile operates, so far, with research financing agencies (see above) and has not contemplated a Ministry of Science and Technology. In 2012, S&T Chile´s investment was of around 0.35% of GDP (*ca*. 700-800 million of US dollars); while the mean S&T investment for OEDC countries was of 2.4% of GDP. The low Chilean investment in S&T means that research strategies need to be thoroughly thought. Hence, it is my view that experience gained along nearly 50 years of well organized and highly respected (by scientists and society) S&T funding system in Chile has paved its academic and societal way to maturity. Part of that has been to maintain a major basic research program (Regular FONDECYT Research Program; with investment of around 50 million of US dollars per year; as well as a S&T Research Strategic Programs, Fellowships, other), based under a predictable and regular financing system. On the other hand, the FONDAP initiative can be highlighted as the most scientifically successful program in Chile and of high world standard. For instance, 85% of FONDAP Centers originated papers that are published in Quartile 1 (Q1) indexed journals and the normalized impact of those publications are about 10% above the world mean. Furthermore, Chile is a leader country in scientific publications (and paper impacts) within the Latin-America subcontinent [9].

Figure 2. Impacts of the February 27th 2010, 8.8 Mw mega-earthquake and tsunami hitting Central Chile [see 11, 12, 13]. **A**. Rock uplifted >2 m in Isla Mocha, Central Chile (former brown intertidal algae can be seen at top of the rock). **B**. Tubul bridge, Central Chile, destroyed by the earthquake. **C & D**. Damages and losses due to tsunami in small-scale artisan boats at Isla Santa María and Tubul villages.

Chile is a country facing numerous, repetitive and dreadful natural and S-IHEs, such as earthquakes and tsunamis (Fig. 2; [11, 12, 13]), volcano eruptions, flooding, drought and large wildfire and landslide events. Therefore, scientific research (preparation, response, recovery, mitigation) and linkages with policy making and government agencies need to be part of integrate interdisciplinary long–term research strategies. The Chilean S&T FONDAP initiative is one of them; and a highly successful one; that perhaps it may be used as a model by other developing or emerging countries. Above all the FONDAP model may be used as a template to incentivize and get going interdisciplinary S&T research of first world class in third world countries. The scientific disciplines (Natural, Cultural and Formal sciences) have to talk-each-other. Moreover, this is a must in developing world countries, where the number of scientists is low and showing rather poor financing schemes.

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Acknowledgments

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