ROYAUME DU MAROC

ACADÉMIE HASSAN II DES SCIENCES ET TECHNIQUES

Perspectives en mathématiques

29-30 octobre 2024

à l'**Académie Hassan II des Sciences et Techniques**

Rabat - Maroc

Sa Majesté le Roi Mohammed VI, que Dieu Le garde,
Protecteur de l'Académie Hassan II
des Sciences et Techniques

PROGRAMME

Journée du mardi 29 octobre 2024

- 09h-9h30 : Bienvenue, allocution du Secrétaire Perpétuel de l'Académie HassanII
  des Sciences et Techniques

- 9h30-10h20 : **Mohamed El Machkouri**, Université Mohammed VI, Plytechnique.

- 10h20-10h50 : Pause.

- 10h50-11h40 : **Mohammed Louriki**, Université Caddi-Ayyad.

- 11h40-12h30 : **Mohamed Ziyat**, Université Mohammed V in Rabat.

- 12h30 : Déjeuner.

- 14h30-15h20 : **Imane Jarni**, Université Mohammed VI Polytechnique.

- 15h20-16h10 : **Youssef Elmadani**, Université Mohammed V de Rabat.

- 16h10-16h40 : Pause.

- 16h40-17h30 : **Salah-Eddine Chorfi**, Université Cadi Ayyad.

Journée du mercredi 30 octobre 2024

- 09h30-10h30 : Conférence, **Mahouton Norbert Hounkonnou,**
  University of Abomey-Calavi, Bénin

- 10h30-11h00 : Pause.

- 11h00-12h30 : Table ronde.

- 12h30 : Clôture

Perspectives in Mathematics

Abstracts

On the Elephant Random Walk and its Variations

Mohamed EL MACHKOURI, Mohammed VI Polytechnic University

The Elephant Random Walk (ERW) was introduced by Schütz and Trimper in [3, 2004] with a view to study memory effects in a one-dimensional discrete-time nearest-neighbor walk on $\mathbb{Z}$ with a complete memory of its whole past. The name of the model is inspired by the traditional saying that elephants can always remember anywhere they have been. The memory of the walker is measured in terms of a parameter p between zero and one and the model exhibits three regimes: diffusive regime ($0 < p < 3/4$), critical regime ($p = 3/4$) and superdiffusive regime ($3/4 < p < 1$). The ERW has drawn a lot of attention in the last years and several theoretical results (law of large numbers, central limit theorem, law of the iterated logarithm,...) have been established for each of the three regimes. In 2022, Gut and Stadtmüller [2] introduced an extension of the ERW model allowing the memory of the walker to gradually increase in time. For this new model, we establish central limit theorems in the three regimes (see [1]) and we show how to estimate the memory parameter p of the model. Finaly, we introduce another new variation of the ERW as a rule for the sequential allocation of drugs in a toxicity-response study.

References

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On the Classification and Decomposition of Stopping Times in an Information-Based Model via Feller Property

Mohammed Louriki, Cadi Ayyad University

We consider an information-based model in which the flow of information about a default time τ is modelled explicitly through the natural completed filtration of a Brownian bridge with length τ. Assuming that the distribution of τ admits a continuous density with respect to the Lebesgue measure, we give the explicit decomposition of any stopping time into totally inaccessible and predictable parts. The resulting conclusion establishes the quasi-left continuity of the underlying filtration and the total inaccessibility of τ. This provides an answer to the open question raised in [1, Problem 3.69, p. 73].

Keywords: Brownian bridge, Markov process, Feller process, totally inaccessible stopping time, predictable stopping time, decomposition of stopping times, credit risk.

References

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Beurling density theorems for sampling and interpolation on a vertical strip

Mohamed Ziyat, Université Mohammed V in Rabat

We consider the Short-Time-Fourier Transform (STFT) with window g, restricted to measurable functions in $\mathbb{R}$, square-integrable in $(0,1)$, and almost periodic with respect to integer translations. The resulting phase space is $\mathbb{C}/\mathbb{Z}$, which is a flat model of an infinite cylinder $\Lambda(\mathbb{Z}) = [0,1) \times \mathbb{R}$. A Gaussian window g leads to the Fock space $\mathcal{F}^{2,\alpha}(\mathbb{C}/\mathbb{Z})$ of $\mathbb{Z}$-quasi-periodic entire functions. We give a complete characterization of the sets $Z \subset \Lambda(\mathbb{Z})$ that are sets of sampling or interpolation, in terms of their upper and lower Beurling densities,

$$D^-(Z) = \liminf_{r \to +\infty} \inf_{w \in \mathbb{C}/\mathbb{Z}} \frac{n(Z, I_{w,r})}{r}, \quad \text{and} \quad D^+(Z) = \limsup_{r \to +\infty} \sup_{w \in \mathbb{C}/\mathbb{Z}} \frac{n(Z, I_{w,r})}{r},$$

where $I_{w,r} = [0,1] \times [\text{Im}(w) - \frac{r}{2}, \text{Im}(w) + \frac{r}{2}]$, and $n(Z, I_{w,r})$ denote the number of points in $Z \cap I_{w,r}$. There is a critical Nyquist density, the real number $\frac{\alpha}{\pi}$, meaning that the condition $D^-(Z) > \frac{\alpha}{\pi}$ characterizes sets of sampling, while the condition $D^+(Z) < \frac{\alpha}{\pi}$ characterizes sets of interpolation. In other words, we obtain a complete characterization of Gabor system $\mathcal{G}(g, Z) = \{e^{2i\pi\xi t}\theta_{\xi,x}(-t, i) : z = (x, \xi) \in Z\}$ as frames and Riesz basic sequences for spaces of functions f, measurable in $\mathbb{R}$, square-integrable in $[0,1]$, and periodic with respect to integer translations.

When g is a Hermite function, we are lead to Bargmann-type transforms into Landau Level eigenspaces on the vertical strip $\mathbb{C}/\mathbb{Z}$. Using the Wexler-Raz orthogonality relations, we obtain a sufficient condition for sampling in these spaces, which is the analogue of a theorem of Gröchenig and Lyubarskii about Gabor frames with Hermite windows.

References

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Reflected McKean-Vlasov SDEs in time-dependent domains

Imane Jarni, Mohammed VI Polytechnic University

In this work, we investigate the deterministic multidimensional Skorokhod problem with normal reflection in a family of time-dependent convex domains that are right continuous left limited with respect to the Hausdorff metric. We then show the existence and uniqueness of solutions to multidimensional McKean-Vlasov stochastic differential equations reflected in these time-dependent domains. Additionally, we derive stability properties with respect to the initial condition and the coefficients. Finally, we establish a propagation of chaos result.

Key word: Reflection; Skorokhod problem; Time-dependent convex region; Reflected MckeanVlasov; Stability.

References

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Recent advances in Dirichlet Spaces

Youssef Elmadani, Mohammed V University in Rabat

Let μ be a positive finite Borel measure on the unit circle in the complex plane, and let $\mathcal{D}(\mu)$ be the associated Dirichlet space defined as follows:

$$\mathcal{D}(\mu) := \left\{ f \in \mathrm{Hol}(\mathbb{D}) : \int_{\mathbb{D}} |f'(z)|^2 P[\mu](z) dA(z) < \infty \right\},$$

where $P[\mu]$ is the Poisson integral of μ and dA is the two-dimensional Lebesgue measure. The Beurling-Deny capacity associated with $\mathcal{D}(\mu)$ is denoted by c_μ. Brown-Shields conjecture for $\mathcal{D}(\mu)$ says that a function $f \in \mathcal{D}(\mu)$ is cyclic for $\mathcal{D}(\mu)$, which means that $\{pf : p \text{ is a polynomial}\}$ is dense in $\mathcal{D}(\mu)$, if and only if f is outer and the boundary zeros set of f is $c_\mu-$capacity zero. In this talk, we introduce a new class of measures for which the Brown-Shields conjecture holds. For these measures, we also provide an explicit characterization of all invariant subspaces for the shift operator. Additionally, we present the representation of outer extremal functions and establish new conditions under which closed sets are $\mu-$polar.

References

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Logarithmic convexity of evolution equations and application to inverse problems

Salah-Eddine Chorfi, Cadi Ayyad University,

In this talk, we present some results on the logarithmic convexity for evolution equations, which is a well-known method in inverse and ill-posed problems. We start with the classical case of self-adjoint operators. Then we analyze the case of analytic semigroups. In this general case, we give an explicit estimate, which will be used to study inverse problems for initial data recovery. We illustrate our abstract result by an application to the Ornstein-Uhlenbeck equations. We discuss both analytic and non-analytic semigroups. We conclude with some recent results for the time-fractional evolution equations with the Caputo derivative of order $0 < \alpha < 1$. We start with symmetric evolution equations. Then, we show that the results extend to the non-symmetric case for diffusion equations provided that the drift coefficient is given by a gradient vector field. We also present some numerical experiments to validate the theoretical results in both symmetric and non-symmetric cases. Finally, some conclusions and open problems will be mentioned.

Keywords: Logarithmic convexity; inverse problem; Ornstein–Uhlenbeck equation; backward problem; fractional evolution equation.

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Title: Geometry and probability on a noncommutative 2-torus in a magnetic field

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Abstract

We describe the geometric and probabilistic properties of a noncomutative $2-$ torus in a magnetic field. We study the volume invariance, integrated scalar curvature, and the volume form by using the operator method of perturbation by inner derivation of the magnetic Laplacian operator on the noncommutative $2-$torus. We then analyze the magnetic stochastic process describing the motion of a particle subject to a uniform magnetic field on the noncommutative 2-torus, and discuss the related main properties.

Keywords: noncommutative 2-torus, magnetic Laplacian, quantum stochastic process.

Mathematics Subject Classification (2010): 46L87; 81S25